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Answers to exam questions

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1 Working with numbers

Using your calculator

Here is an exam question ...



Work out the following. Give your answers to 2 decimal places.

- a 4.2^4 [1]
 b $\frac{3.9^2 + 0.53}{3.9 \times 0.53}$ [2]
 c 350×1.005^{12} [1]

... and its solution

a 311.17

Key in

311.1696

b 7.61

Key in

7.614900...

c 371.59

Key in

371.587234...

Now try these exam questions



- 1 Work out these, giving the answers to 2 decimal places.
- a 3.4^5 [1]
 b $(5.1 + 3.7) \times 4.2$ [1]
 c $\frac{5.1 \times 2.6}{14.2 - 6.3}$ [2]
- 2 Work out the reciprocal of each of these. Give your answers to 2 decimal places where appropriate.
- a 50 [1]
 b 0.75 [1]
 c 3^2 [1]

3 Work out these.

- a $\frac{3}{5}$ of 200 g [1]
 b $2\frac{3}{4} - 1\frac{4}{5}$ [2]
 c $\frac{4}{7}$ of £26.60 [1]

4 Work out these, giving your answers to 2 decimal places where appropriate.

- a 730×1.01^{15} [1]
 b $14^{\frac{1}{3}}$ [1]
 c $\frac{840 \times 1.03}{840 + 1.03}$ [2]

More exam practice

1 Work out these.

- a 3.4^2 [1]
 b The reciprocal of $2\frac{1}{2}$ [1]
 c $\sqrt[3]{729}$ [1]

2 Work out these.

- a $4\frac{2}{3} + 2\frac{3}{4}$ [2]
 b $2\frac{4}{7} - 1\frac{2}{3}$ [2]
 c $\frac{3}{5} \times \frac{3}{7}$ [2]

3 Work out these.

- a 5^6 [1]
 b $\sqrt{31}$ (Give the answer to 2 d.p.) [1 + 1]
 c $\frac{3.84}{2.19 - 1.59}$ [1]

4 Jo invests £10 000 in a two-stage bond.

Jo uses the following calculator to find how much her bond will be worth after 6 years.

$$10\,000 \times (1.045)^4 \times (1.065)^2$$

Work this out correct to the nearest pound. [2 + 1]

Accuracy of answers

Here is an exam question ...

There are 4.54609 litres in a gallon.

Round 4.54609

- a to 3 decimal places. [1]
 b to 3 significant figures. [1]

... and its solution

- a 4.546
 b 4.55

Now try this exam question

- 1 The attendance at a world cup match was 54 682. The TV commentator said that there were 55 000 people. To how many significant figures was the commentator's number? [1]

More exam practice

- 1 a Explain why 2.974 correct to 1 significant figure is not 3.0. [1]
 b Write 73.14 correct to 1 significant figure. [1]
- 2 Joanne got the answer 146.971 24 to a problem on her calculator. Write 146.971 24 correct to
 a the nearest whole number. [1]
 b 1 significant figure. [1]
 c 1 decimal place. [1]
- 3 a Write 1.8047 correct to 2 decimal places. [1]
 b Write 1048 correct to 3 significant figures. [1]

Prime numbers and factors

Here is an exam question ...

- a Find the highest common factor and lowest common multiple of 12 and 16. [4]
- b Work out these, writing each answer as a whole number.
- i $5^6 \div 5^4$ [1]
 ii $2^3 \times 2^5 \div 2^7$ [1]
 iii $6^2 \times 5^2 \div 2^2$ [2]

... and its solution

- a $12 = 2 \times 2 \times 3$
 $16 = 2 \times 2 \times 2 \times 2$
 $\text{HCF} = 2 \times 2 = 4$ Two 2s are common to both.
 $\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 = 48$ Four 2s and one 3 are in at least one of the numbers.
- b i $5^6 \div 5^4 = 5^2 = 25$ $6 - 4 = 2$
 ii $2^3 \times 2^5 \div 2^7 = 2^1 = 2$ $3 + 5 - 7 = 1$
 iii $6^2 \times 5^2 \div 2^2 = 36 \times 25 \div 4 = 225$

Chief Examiner Says

6, 5 and 2 are different numbers so do not try to collect the indices.

Now try these exam questions

- 1 Write the following as whole numbers.
 a 2^6 [1]
 b 5^3 [1]
 c $4^5 \times 4^2 \div 4^3$ [2]
- 2 a Write 30 as the product of its prime factors. [2]
 b Write down the prime factor of 30 that is also a prime factor of 21. [1]
- 3 Find the highest common factor and lowest common multiple of 10, 12 and 20. [5]

More exam practice

- 1 a Write 24 as the product of its prime factors. [2]
 b Find the lowest common multiple of 24 and 60. [2]
- 2 a Write 48 as the product of its prime factors. [2]
 b Find the highest common factor of 48 and 72. [2]
- 3 Explain how you know that 83 is the only prime number between 80 and 88. [3]

2 Algebra

Here is an exam question ...

- a Expand the brackets and write as simply as possible $2(3x - 4) - 5(x + 3)$. [2]
 b Factorise completely $3a^2 + 6ab$. [2]

... and its solution

- a $6x - 8 - 5x - 15 = x - 23$.
 b $3a(a + 2b)$ Take care with the signs. $-5x + 3 = -15$
3a is common to both terms.

Now try these exam questions

- 1 Expand $5s(s^2 - 2)$. [2]
 2 Multiply out and simplify $5(a + 2) - 3(a - 1)$. [2]
 3 Factorise completely $4x^2 - 2x$. [2]

More exam practice

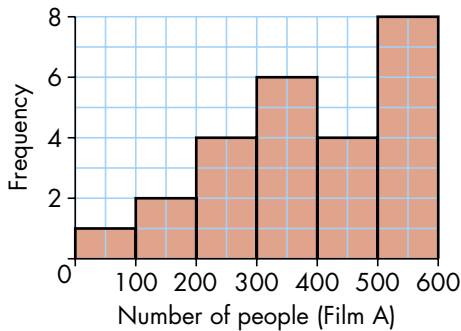
- 1 Expand $5x(x - 3)$. [2]
 2 Expand $3a(2a + 4)$. [2]
 3 Multiply out and simplify $2(3x + 1) - 4(x - 3)$. [2]
 4 Factorise.
 a $5a - 10b$ [1]
 b $x^2 + 7x$ [1]

3 Statistical diagrams

Here is an exam question ...

The manager of the Metro cinema records the numbers of people watching each of two films for 25 days. The frequency diagram is for Film A.

The table shows the numbers of people who watched Film B. Compare the two distributions. [2]



Number of people (Film B)	Frequency
0 – 99	5
100 – 199	12
200 – 299	6
300 – 399	1
400 – 499	0
500 – 599	1

... and its solution

The average attendance at Film A was much higher, or more people watched Film A. The numbers attending Film A were more varied or the number watching Film B each night was more consistent.

Now try these exam questions

1 A class of 33 students sat a mathematics exam. Their results are listed below.

89 78 56 43 92 95 24 72 58
65 55 98 81 72 61 44 48 76
82 91 76 81 74 82 99 21 34
79 64 78 81 73 69

- Draw an ordered stem-and-leaf diagram for this information. [3]
- Find the median mark. [1]

2 A botanist collected samples of leaves from two oak trees. The stem-and-leaf diagrams show the lengths in centimetres.

Tree A

```

5 | 5 7
6 | 0 2 2 3 5 7 8
7 | 1 3 8 9
8 | 2 2 3 8
9 | 4 5 7
    
```

Tree B

```

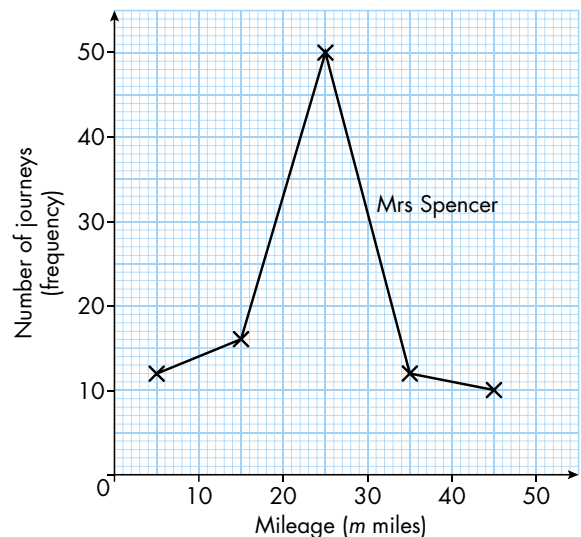
5 |
6 | 4 5 6 6 7 9
7 | 0 0 1 2 4 5 5 6 7 8
8 | 1 1 3 7
9 |
    
```

Key: 6 | 4 = 6.4 cm

Compare the two distributions. [2]

More exam practice

- Mrs Spencer and Mr Patel both work for the same company. In 2010 they each recorded the mileage of every journey they made for the company. The mileages for Mrs Spencer's journeys are summarised in the frequency polygon.



The mileages for Mr Patel's journeys are summarised in this table.

Mileage (m miles)	Frequency
$0 < m \leq 10$	38
$10 < m \leq 20$	44
$20 < m \leq 30$	10
$30 < m \leq 40$	8

- a Copy the diagram and draw on it the frequency polygon for the mileages of Mr Patel's journeys. [2]
- b Make two comparisons between the mileages of Mrs Spencer's and Mr Patel's journeys. [2]
- 2 This table summarises the heights, h cm, of the students in Class 11A.

Height h cm	Frequency
$140 < h \leq 150$	2
$150 < h \leq 160$	5
$160 < h \leq 170$	13
$170 < h \leq 180$	7
$180 < h \leq 190$	3

Draw a frequency polygon to show these results. [2]

4 Equations

Here is an exam question ...

Solve the following equations.

- a $2(3 - x) = 1$ [2]
- b $\frac{5x + 8}{3} = 6$ [3]
- c $4(x + 7) = 3(2x - 4)$ [4]

... and its solution

- a $6 - 2x = 1$
 $-2x = -5$
 $x = 2\frac{1}{2}$
- b $\frac{5x + 8}{3} = 6$
 $5x + 8 = 18$
 $5x = 10$
 $x = 2$
- c $4(x + 7) = 3(2x - 4)$
 $4x + 28 = 6x - 12$
 $40 = 2x$
 $x = 20$

Now try these exam questions

- 1 Solve $\frac{3m}{4} = 9$. [2]
- 2 Solve $2(y + 3) = 5y$. [3]
- 3 Solve $4(x + 2) + 2(3x - 2) = 14$. [4]

More exam practice

- 1 Solve $5(x - 2) = 25$. [3]
- 2 Solve $4(a + 3) = 18$. [3]
- 3 Solve $3x = x + 1$. [2]
- 4 Solve $3p - 4 = p + 8$. [3]
- 5 Solve $4(x - 1) = 2x + 3$. [3]

5 Ratio and proportion

Here is an exam question ...

John and Peter did some gardening. They shared the money they were paid in the ratio of the number of hours they worked.

John worked for 5 hours. Peter worked for 7 hours. They were paid a total of £28.80.

How much did each one receive? [2]

... and its solution

Ratio is 5:7

Total = 12

One share = $28.8 \div 12$
 $= £2.40$

John receives $5 \times 2.40 = £12$

Peter receives $7 \times 2.40 = £16.80$

Check: $£12 + £16.80$
 $= £28.80$

Now try these exam questions

- 1 Some of the very first coins were made with three parts silver to seven parts gold.
- a How much gold should be mixed with 15 g of silver in one of these coins? [2]
- b Another coin made this way has a mass of 20 g. How much gold does it contain? [2]
- 2 A recipe for rock cakes uses 100 g of mixed fruit and 250 g of flour. This makes 10 rock cakes. Jason wants to make 25 rock cakes. How much mixed fruit and flour does he need? [2]
- 3 A car park contains vans and cars. The ratio of vans to cars is 1:6. There are 420 vehicles in the car park.
- a How many vans are there?
- b How many cars are there? [2]
- 4 Adrian, Penelope and Gladys shared a lottery win in the ratio 2:5:8. They won £7000. How much did each receive, correct to the nearest penny? [3+1]
- 5 The table shows the prices of different packs of chocolate bars.

Pack	Size	Price
Standard	500 g	£1.15
Family	750 g	£1.59
Special	1.2 kg	£2.49

Find which pack is the best value for money. You must show clearly how you decide. [4]

- 6** In February 2007 the exchange rate between pounds and US dollars was £1 = \$1.93. Liz changed £500 into US dollars. She went on holiday and spent \$784. She changed the rest back into pounds. How much did she receive in pounds? [4]
- 7** Coverswift charges £27.50 for five days' holiday insurance and an extra £4.50 for every day after that. How much did they charge for a 14-day holiday? [3]

More exam practice

- 1** Pewter is made from tin and lead in the ratio 4:1. A pewter mug has a mass of 250 g. How much lead does it contain? [2]
- 2** Helen, Jessica and Rebecca are left £13 500 to be shared in the ratio of their ages. They are 16, 19 and 25 years old. How much does each get? [3]
- 3** The Winder and the Gower families had a meal together. They agreed to share the cost in the ratio of the sizes of their families, which is 2:3. The Gower family paid £21.60. What was the total bill? [3]
- 4** Eau de Parfum is made from three parts flower extract, eight parts water and nine parts alcohol.
- a** How much water does a 250 ml bottle of Eau de Parfum contain? [2]
- b** How many litres of Eau de Parfum can be made with 45 litres of flower extract? [2]
- 5** The ratio of lorries to vans to cars passing a point on a motorway is 2:3:7. In one hour, 6492 vehicles passed the point. How many of these vehicles were lorries? [2]
- 6** Three bottles of mineral water were for sale at a supermarket.
- 3 litres for £1.50
2 litres for £1.10
750 ml for 37p
- Which bottle was the best buy? [4]

6 Statistical calculations

Here is an exam question ...

A wedding was attended by 120 guests. The distance, d miles, that each guest travelled was recorded in the frequency table.

Distance (d miles)	Number of guests
$0 < d \leq 10$	26
$10 < d \leq 20$	38
$20 < d \leq 30$	20
$30 < d \leq 50$	20
$50 < d \leq 100$	12
$100 < d \leq 140$	4

Calculate an estimate of the mean distance travelled. [4]

... and its solution

Use the mid-interval values, 5, 15, ... to represent each group.

$$\begin{aligned} \text{Mean} &= \frac{5 \times 26 + 15 \times 38 + \dots + 120 \times 4}{120} \\ &= 3380 \div 120 \\ &= 28.2 \text{ miles} \end{aligned}$$

Now try this exam question

- 1** The number of matches in each of 100 boxes was counted. Here are the results.

Number of matches	45	46	47	48	49	50
Frequency	14	25	32	19	8	2

Calculate the mean number of matches in a box. [3]

More exam practice

- 1** This table summarises the heights, h cm, of the students in Class 11A.

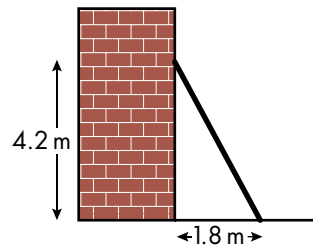
Height (h cm)	Frequency
$140 < h \leq 150$	2
$150 < h \leq 160$	5
$160 < h \leq 170$	13
$170 < h \leq 180$	7
$180 < h \leq 190$	3

Calculate an estimate of the mean height of the class. [4]

- 2 150 apples were picked from an orchard and weighed. Their masses are shown in the table. Calculate an estimate of the mean mass of an apple. [3]

Mass (m grams)	Number of apples	Mid-interval value
$50 < m \leq 60$	23	55
$60 < m \leq 70$	42	
$70 < m \leq 80$	50	
$80 < m \leq 90$	20	
$90 < m \leq 100$	15	

- 2 Calculate the length of this ladder. [3]



Try this exam question

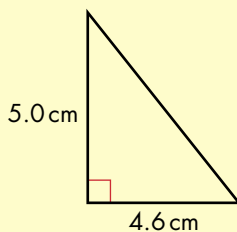
- 1 Find the midpoint and length of the line joining the points $A(5, 4)$ and $B(8, -2)$.

7 Pythagoras' theorem

Using Pythagoras' theorem

Try these exam questions

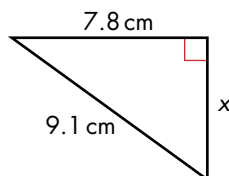
- 1 Calculate the length of the hypotenuse of this triangle. Give your answer to a sensible degree of accuracy.



- 2 Jane has a pencil box measuring 20 cm by 5 cm by 3 cm. Can a pencil 23 cm long fit in the box? Show how you decide. [3]

More exam practice

- 1 Find the length of the side marked x .



[3]

8 Formulae 1

Here is an exam question ...

The price of a handtool of size S cm is P pence. The formula connecting P and S is $P = 20 + 12S$.

- a Calculate the price of a handtool of size 3 cm. [2]
 b Calculate the size of a handtool whose price is 95p. [2]
 c Rearrange the formula $P = 20 + 12S$ to express S in terms of P . [2]

... and its solution

$$\begin{aligned} \text{a } P &= 20 + 12 \times 3 \\ &= 20 + 36 \\ &= 56 \end{aligned}$$

The price is 56p

$$\begin{aligned} \text{b } 20 + 12S &= 95 \\ 12S &= 75 \\ S &= 75 \div 12 \\ &= 6.25 \end{aligned}$$

The size is 6.25 cm

$$\begin{aligned} \text{c } P - 20 &= 12S \\ S &= \frac{P - 20}{12} \end{aligned}$$

Now try these exam questions

- 1 Using $u = 9$, $t = 48$ and $a = \frac{1}{4}$, work out the value of s from the formula $s = ut + \frac{1}{2}at^2$. [3]
 2 Use the formula $F = 2(C + 15)$ to find an expression for C in terms of F . [3]
 3 Rearrange the following to give d in terms of e . $e = 5d + 3$ [2]

More exam practice

- The cost, C pence, of printing n party invitations is given by $C = 120 + 4n$.
Find a formula for n in terms of C . [2]
- Make y the subject of the formula $5x + 6y = 30$. [2]
- The formula $f = \frac{uv}{u+v}$ is used in the study of light. Calculate f when $u = 14.9$ and $v = -10.2$.
Give your answer to 3 significant figures. [2]
- For each of the following, state whether it is an expression, formula, equation or identity.
 - $5x^2 - 2x = x(5x - 2)$ [1]
 - $5x^2 - 2x$ [1]
 - $C = 3n + 5$ [1]
 - $3n + 5 = 7$ [1]
- Complete the boxes so that this is an identity.
 $15x - 7 \equiv 8x + \square x - 5 + \square$ [2]
- You are given that
 $3a(a - 7) + ca + d = 3a^2 + 10a + 1$
Find the values of c and d that make this an identity. [3]

9 Measures

Here is an exam question ...

For each of these, write the most suitable metric unit to use for measuring.

- The length of a football pitch [1]
- The amount of liquid that a teaspoon can hold [1]
- The area of a square with side 5 cm [1]

... and its solution

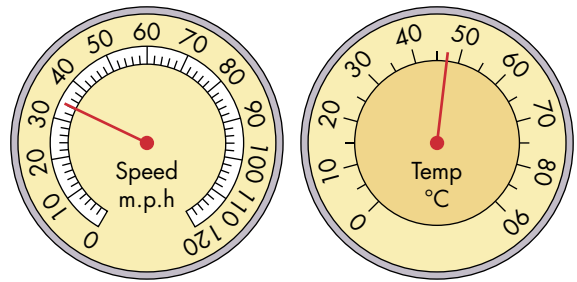
- metres (m)
- millilitres (ml)
- square centimetres (cm²)

Now try these exam questions

- Pat weighs 106 pounds. Estimate her weight in kilograms.
 - She is 5 feet tall. How tall is this in metres? [4]
- Ben pours five 33 ml drinks out of a full bottle containing 1.5 litres. How much drink is left in the bottle? [2]
- P is 8 km from O on a bearing of 037° and Q is 7 km due east of O.
 - Make a scale drawing showing O, P and Q. Use a scale of 1 cm to 2 km. [3]
 - Find the distance between P and Q. [1]
 - Find the bearing of P from Q. [1]

More exam practice

- Estimate the height in metres of a double decker bus. [1]
 - Two of the instruments on the bus are shown below.



- What speed is shown? [1]
 - What temperature is shown? [1]
- An electrician used two pieces of wire, each 160 cm long.
 - Find the total length of wire he used. [1]
He cut the two pieces from a 10 m length of wire.
 - How much wire did he have left over? State clearly the units of your answer. [2]
 - Trevor has 20 metres of string. He uses three pieces to tie some parcels. The pieces are 125 cm, 220 cm and 240 cm long. How much string is left? State the units of your answer. [3]
 - An ordinary wine bottle holds 750 ml. A display bottle holds the same as 20 ordinary bottles. How much wine does the display bottle hold? Give your answer in litres. [3]

10 Planning and collecting

Here is an exam question ...

Here are two questions that Amy wanted to include in a survey she was going to do to find out if people liked the new shopping centre in her town.

Rewrite each one to show how you would word the questions in a questionnaire and why you would change it.

- How old are you? [2]
- This new shopping centre appears to be a success. Do you agree? [2]

... and its solution

Possible answers would be:

- a The question may be thought to be personal – some people may not answer.

Change to:

What is your age? Tick the appropriate box.

10–19 20–29 30–39
 40–49 50–59 60 or over

- b This is a leading question. Change to:

Do you think the shopping centre is a success?
 Tick the appropriate box.

yes no don't know

Now try this exam question

- 1 Wasim, Abbie and Oliver carried out a survey to find the number of cars passing their school in ten-minute intervals over a period of six hours. The results are shown below.

14	21	30	26	51	39	31	15	8
17	20	16	18	25	16	34	28	26
7	12	5	44	36	58	25	22	24
17	42	13	44	28	33	16	27	26

To analyse their results they each decided to group their data and make a frequency table.

Wasim chose these groups:

0–19, 20–39, 40–59.

Abbie chose these groups:

0–10, 10–20, 20–30, 30–40, 40–50, 50–60.

Oliver chose these groups:

0–9, 10–19, 20–29, 30–39, 40–49, 50–59.

Their teacher said that Oliver's was the best method.

- a Give one reason each for why she thought Wasim's and Abbie's groups were unsuitable. [2]
 b Copy and complete the following frequency table using Oliver's groups of numbers of cars. You may want to use tally marks.

Cars	Frequency
0–9	
10–19	
20–29	
30–39	
40–49	
50–59	

[2]

More exam practice

- 1 Pali did a survey about school meals. He included the following questions amongst others. State one thing that is wrong with each question.

a Don't you think they should serve fish on Fridays? [1]

b Would you like to see more salads and more burgers? [1]

11 Sequences

Here is an exam question ...

- a These are the first four terms of a sequence.

2, 9, 16, 23

i Write down the term-to-term rule. [1]

ii Find the sixth term of this sequence. [1]

- b These are the first four terms of a sequence.

29, 25, 21, 17

i Find the seventh term. [1]

ii Explain how you worked out your answer. [1]

- c Here is the term-to-term rule for another sequence.

Multiply the previous term by 4 then subtract 1.

The first term of the sequence is 2.

Find the third term. [1]

... and its solution

- a i the rule is +7

ii 37 $23 + 7 + 7 = 37$

- b i 5

ii The term-to-term rule is -4 ; $17 - 4 - 4 - 4 = 5$

- c 27 $2 \times 4 - 1 = 7, 7 \times 4 - 1 = 27$

Here is another exam question ...

- a Write down the 10th term for the sequence

3, 7, 11, 15, ... [1]

- b Write down an expression for the n th term. [2]

- c Show that 137 cannot be a term in this sequence. [3]

... and its solution

- a 10th term = $3 + 9 \times 4 = 39$

- b Either:

the difference between terms is 4 so the expression will start $4n$.

If $n = 1$ then $4n = 4$

subtract 1 to get 3

therefore the expression is $4n - 1$.

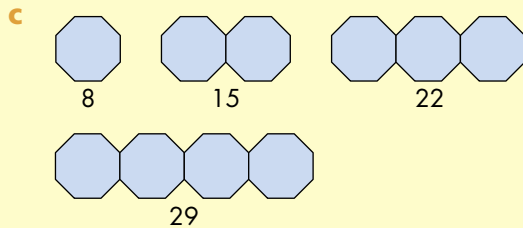
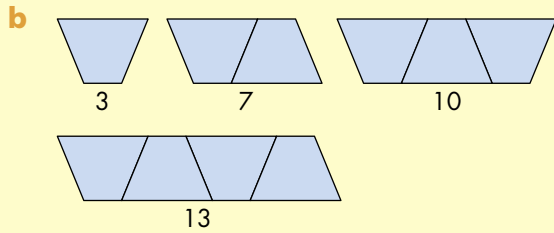
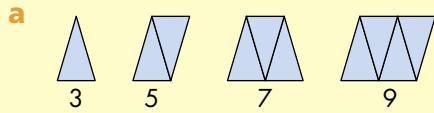
Or:

1st term is 3, add 4 ($n - 1$) times therefore n th term is $3 + 4(n - 1) = 3 + 4n - 4 = 4n - 1$.

- c If 137 is in the sequence then $4n - 1 = 137$
 $4n = 138$
 $n = 138 \div 4 = 34.5$
 i.e. not a whole number.
 Therefore 137 cannot be in the sequence.

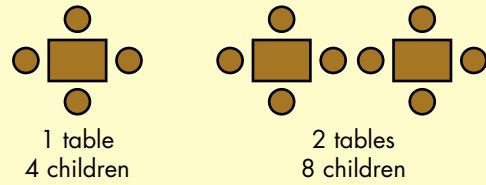
Now try these exam questions

- 1 For each of these sequences, the numbers are the number of lines in each picture.



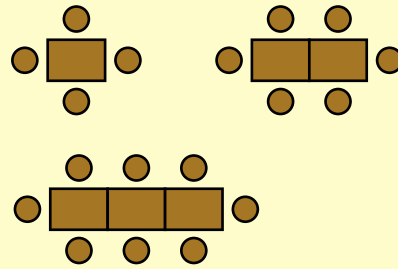
- i Draw the next two pictures in each of the sequences. [1][1][1]
 ii Explain what you need to do to the previous number to get the next number. [1][1][1]
- 2 The sequence below starts 1, 2, 1. The next term is the previous three terms added together.
 1, 2, 1, 4, 7, 12, 23, ...
- a Write down the next two terms of the sequence. [2]
 b There seems to be another pattern in this sequence involving odd and even numbers.
 1 (odd), 2 (even), 1 (odd), 4 (even), ...
 Does this 'odd, even' pattern continue for the next few numbers? [1]
 Give examples to support your answer. [2]
- 3 The first four terms of a sequence are 2, 9, 16, 23.
- a Find the rule for this sequence. [2]
 b Show that 300 is not in this sequence. [3]

- 4 These classroom tables seat four children.



- a For this arrangement, write down a formula linking the number of children, c , to the number of tables, t . [1]

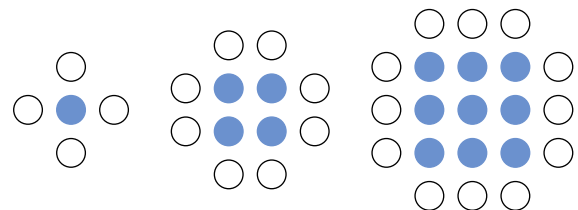
- b Sometimes the tables are arranged like this:



- For this arrangement, write down a formula linking the number of children, c , to the number of tables, t . [2]

More exam practice

- 1 For each of these sequences:
- i write down the next two terms of the sequence. [1 + 1 + 1]
 ii write down the term-to-term rule for the sequence. [1 + 1 + 1]
- a 1, 6, 11, 16, 21, ...
 b 18, 15, 12, 9, ...
 c 1, 3, 9, 27, ...
- 2 The first two terms of a sequence are 1 and 5.
- a i How could the sequence continue? Write down the next three numbers. [1]
 ii Write down the rule you have used. [1]
 b i Find another way to continue the sequence. Write down the next three numbers. [1]
 ii Write down the rule you have used. [1]
- 3 Here is a sequence of patterns.



- a Draw the next pattern. [1]
 b Complete the table. [2]

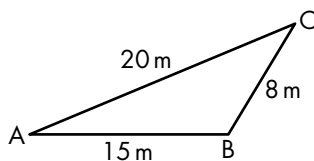
Pattern number	1	2	3	4
Number of black dots	1	4		
Number of white dots	4			

- c Describe the sequence of black dots. [1]
 d Describe the sequence of white dots. [1]
 4 The first four terms of a sequence are 3, 8, 13, 18. Find the 50th term and the rule for this sequence. [3]

12 Constructions and loci

Here is an exam question ...

- a Two buoys are anchored at A and B. B is due East of A. A boat is anchored at C.



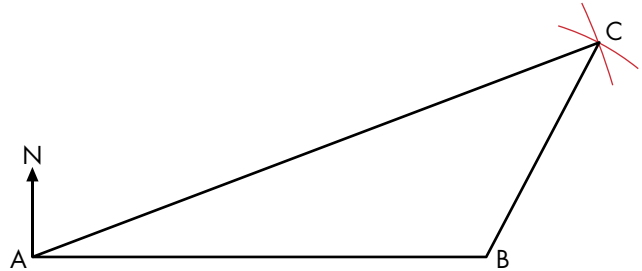
- i Using a scale of 1 cm to 2.5 m, draw the triangle ABC. [2]
 ii Measure the bearing of the boat, C, from buoy A. [2]
 b This is the plan of a garden drawn on a scale of 1 cm to 2 m.



A pond is to be dug in the garden. The pond must be at least 4 m from the tree. It must be at least 3 m from the house. Shade the region where the pond can be dug. Show all your construction lines. [3]

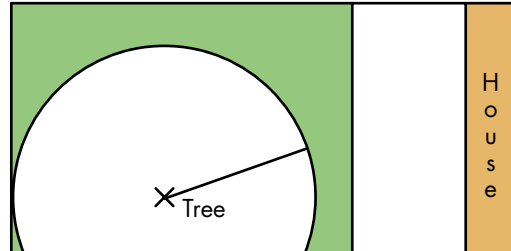
... and its solution

- a i **Step 1:** Draw the line AB 6 cm long.
Step 2: Using compasses, draw an arc 8 cm from A, and an arc 3.2 cm from B.
Step 3: Mark the point C where the arcs cross and join to A and B to complete the triangle.
 ii To measure the bearing, use your protractor, to draw the North line at A, at right-angles to AB. Now use your protractor, with the zero line along the North line, to measure the bearing. It should be between 069° and 070° .



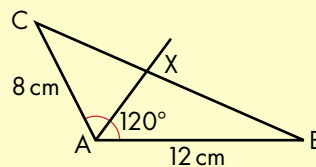
- b At least 4 m from the tree means it is outside a circle radius 2 cm, centre the tree. At least 3 m from the house means it is to the left of a line parallel to the house and 1.5 cm from it. The pond can be dug in the region shaded green.

Scale 1 cm to 2 m



Now try these exam questions

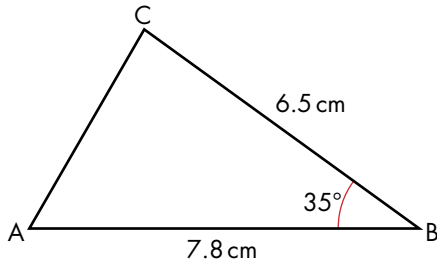
- 1 The diagram shows a triangle ABC. The bisector of the angle at A meets line BC at X.



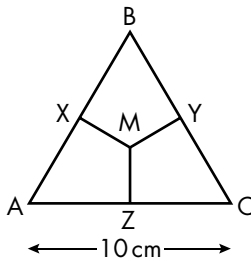
- a Construct the triangle and the bisector of angle A. [4]
 b Measure the distance AX. [1]
 2 Ashwell and Buxbourne are two towns, 50 km apart. Chris is house-hunting. He has decided he would like to live closer to Buxbourne than Ashwell but no further than 30 km from Ashwell. Using a scale of 1 cm to represent 5 km, construct and shade the area in which Chris should look for a house. [4]

More exam practice

- 1 Here is a sketch of a triangle.



- a Make an accurate drawing of the triangle.
b Measure the length AC.
- 2 ABC is an equilateral triangle of side 10 cm. MX is the perpendicular bisector of side AB. MY is the perpendicular bisector of side BC. MZ is the perpendicular bisector of side AC.

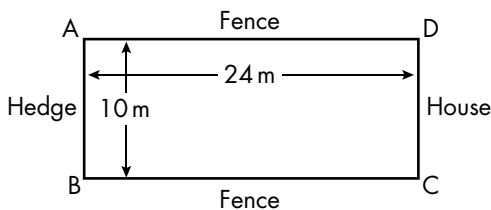


Construct accurately the triangle and the perpendicular bisectors.

Measure the distance MZ.

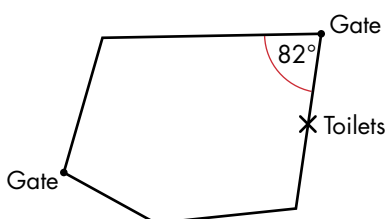
- 3 Ashad's garden is a rectangle. He is deciding where to plant a new apple tree.

It must be nearer to the hedge AB than to the house CD. It must be at least 2 m from the fences AC and BD. It must be more than 6 m from corner A.



Shade the region where the tree can be planted. Leave in all your construction lines. Make the scale of your drawing 1 cm to 2 m.

- 4 Sandtown council wants to build a pavilion on a playing field. They decide to site it at an equal distance from both gates and not more than 250 m from the toilet block.



To answer this question, first draw a pentagon of about this shape, with the angle at the top, as indicated, of size 82° .

Make the gates 13.2 cm apart, and the toilet block 4.5 cm from one gate.

On a scale drawing, make suitable constructions to locate the site. Leave in your construction lines, and show clearly where the pavilion can be built. Use a scale of 1 cm to represent 50 m.

[2]
[1]

13 Sampling

Here is an exam question ...

A homework survey is to be carried out in a large school of 1700 students. Because of the school's size, it is impractical to obtain the view of each student. You decide to sample the students to obtain an accurate picture of times spent on homework.

List three steps you would take to ensure that you would obtain a representative sample.

[6]

... and its solution

- 1 Make sure sample is big enough (e.g. 10%).
 - 2 Make sure all ages/year groups are represented (e.g. stratified).
 - 3 Make sure both genders are represented.
- Or some description of random sampling.

Chief Examiner Says

Make sure you read the question carefully.

This question is about the sample, not about the questionnaire, conditions of questioning, etc.

Now try these exam questions

- 1 Fiona, Raiza and Simon conduct a survey on the way students travel to their school. To do this they each decide to take a 10% sample. The school has 800 students.
 - a Give one advantage and one disadvantage of using a sample to obtain the data. [2]
 - b Fiona decides to go outside the school gate at 8.45 am and ask the first 80 students who arrive. Give one reason why this is not a good way to obtain the sample. [1]
 - c Raiza decides to take a simple random sample of 10% of the students. Describe a way in which Raiza might select her random sample. [2]
 - d Simon decides to take a stratified random sample. Give one advantage this may have over a simple random sample and suggest possible strata. [2]

2 You are doing a survey about kitchen appliances. You are going to sample the households in a small town. You decide to take a stratified sample of 10% of the households.

- a** Describe carefully how you would choose your stratified sample. [2]
b State one advantage your method has over a simple random sample [1]
c State one advantage your method has over taking every 10th name from the local telephone directory. [1]

More exam practice

1 Michael carried out a survey on the number of hours spent on homework in a week. He asked 90 students in his school.

Micheal used simple random sampling to choose the 90 students.

a Describe carefully a simple random sampling method he could have used. [2]

Abigail also carried out a homework survey at her school.

She chose, at random, 10% of each year group.

b Name the type of sampling method that Abigail used. [1]

c Give one advantage that Abigail's method has over Michael's method. [1]

d Give one advantage that Michael's method has over Abigail's method. [1]

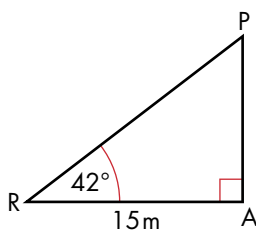
2 a Describe the difference between a sample of a population and a census of a populaton. [2]

b Give one advantage and one disadvantage of each. [4]

14 Trigonometry

Here is an exam question ...

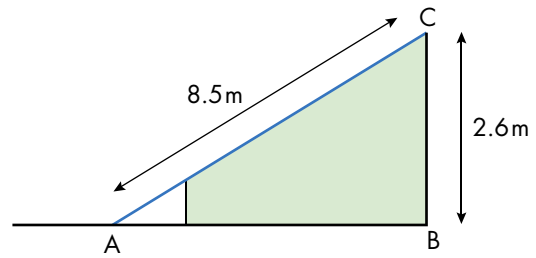
a AP is a telegraph pole.



The angle of elevation of the top of the pole from a point B on the ground is 42° .

Calculate the height of the pole. [3]

b A slide in an open-air swimming pool is built down the side of a bank. The slide is 8.5 m long and descends 2.6 m from the top to the water level.



i Calculate the distance AB. [2]

ii Calculate the angle that the slide makes with the horizontal AB [3]

... and its solution

$$\mathbf{a} \quad \tan 42^\circ = \frac{AP}{AB} = \frac{AP}{15}$$

$$\text{Therefore } AP = 15 \times \tan 42^\circ$$

$$AP = 13.5$$

The pole is 13.5m high.

$$\mathbf{b} \quad \mathbf{i} \quad AC^2 = AB^2 + BC^2$$

$$8.5^2 = AB^2 + 2.6^2$$

$$AB^2 = 8.5^2 - 2.6^2$$

$$= 65.49$$

$$AB = 8.1$$

The distance AB is 8.1m.

$$\mathbf{ii} \quad \sin A = \frac{O}{H}$$

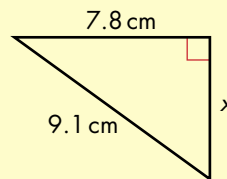
$$= \frac{2.6}{8.5}$$

$$= 0.3059$$

$$\text{Angle } A = 17.8^\circ$$

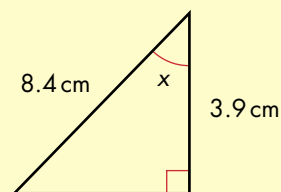
Now try these exam questions

1 a Find the length of the side marked x .



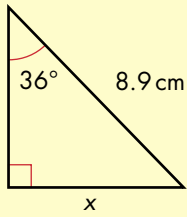
[3]

b Find the angle marked x .

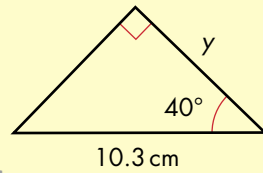


[3]

2 Calculate the lengths marked with letters.



[3]

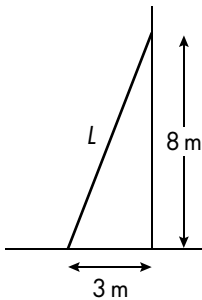


[3]

15 Representing and interpreting data

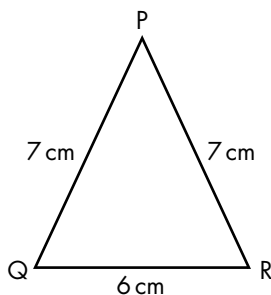
More exam practice

1 A ladder of length L is placed against a vertical wall. The ground is horizontal, the foot of the ladder is 3 m from the wall and the top is 8 m vertically above the ground.

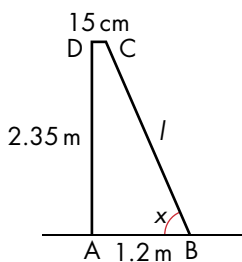


- a Find the length of the ladder. [2]
- b The foot of the ladder is moved a further 2 m from the base of the wall. Find the distance the ladder moves down the wall. [3]
- c Calculate how far up the wall the ladder reaches when it makes an angle of 75° with the ground. [3]

2 Find the angle PQR.



3 ABCD is the side of the frame of the goal used by a football club.



- a Calculate the length of the sloping strut, l , joining C to B. [3]
- b Calculate the angle, x , that CB makes with the ground. [3]

Here is an exam question ...

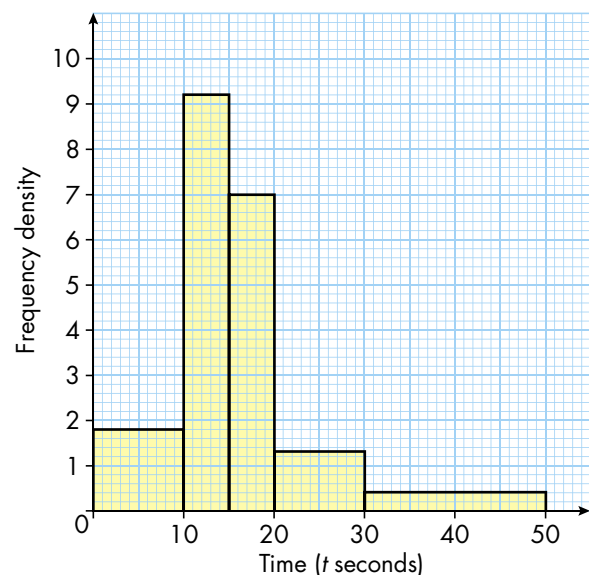
A test was carried out to establish the ability of a mouse to find food. The test was carried out on 120 mice. The distribution of times taken to reach the food is given in the table.

Time (t seconds)	Frequency
$0 < t \leq 10$	18
$10 < t \leq 15$	46
$15 < t \leq 20$	35
$20 < t \leq 30$	13
$30 < t \leq 50$	8

Draw a histogram to represent this information. [4]

... and its solution

Time (t seconds)	Frequency	Frequency density
$0 < t \leq 10$	18	1.8
$10 < t \leq 15$	46	9.2
$15 < t \leq 20$	35	7
$20 < t \leq 30$	13	1.3
$30 < t \leq 50$	8	0.4



Now try these exam questions

- 1 The weight loss in pounds of each of 80 members of a slimming club is summarised in the table.

Weight loss (w pounds)	Frequency
$0 < w \leq 5$	9
$5 < w \leq 10$	13
$10 < w \leq 15$	21
$15 < w \leq 20$	17
$20 < w \leq 25$	10
$25 < w \leq 30$	8
$30 < w \leq 35$	2

- a i Copy and complete the cumulative frequency table. [1]

Weight loss (w pounds)	Cumulative frequency
$w \leq 5$	9
$w \leq 10$	
$w \leq 15$	
$w \leq 20$	
$w \leq 25$	
$w \leq 30$	
$w \leq 35$	

- ii Draw the cumulative frequency graph for the weight loss of the club. Use a scale of 2 cm to 5 pounds on the horizontal axis and 1 cm to 10 members on the vertical axis. [3]

- b Use your graph to find the following. [1]
 i The median weight [1]
 ii The interquartile range [2]
 c The company running the club guarantees that everyone will lose at least 8 pounds. Use your graph to estimate how many members achieve this target. [2]
 d Draw a box plot to summarise the distribution. [3]

- 2 A doctor's patients are grouped by ages as shown in the table.

Age (x) in years	Number of calls
$0 < x \leq 5$	14
$5 < x \leq 15$	41
$15 < x \leq 25$	59
$25 < x \leq 45$	70
$45 < x \leq 75$	16

- Draw a histogram to represent this information. Use a scale of 2 cm to 10 years on the horizontal axis and an area of 1 cm^2 to represent five patients. [3]

- 3 A hospital recorded the mass at birth, in kilograms, of 100 boys and 100 girls. The results are summarised in the table.

	Girls	Boys
Median	3.2	3.3
Lower quartile	2.6	2.3
Upper quartile	3.8	4.0
Minimum	1.2	1.2
Maximum	4.4	4.7

- a Draw two box plots to show this information. [6]
 b Use your diagrams to compare the mass at birth of boys and girls. [2]

More exam practice

- 1 Gary recorded the playing times of each track in his CD collection. The table shows the grouped distribution times.

Times (t minutes)	Number of tracks (frequency)
$1 < t \leq 2$	5
$2 < t \leq 3$	25
$3 < t \leq 4$	45
$4 < t \leq 5$	82
$5 < t \leq 6$	33
$6 < t \leq 7$	10

- a i Copy and complete this cumulative frequency table. [1]

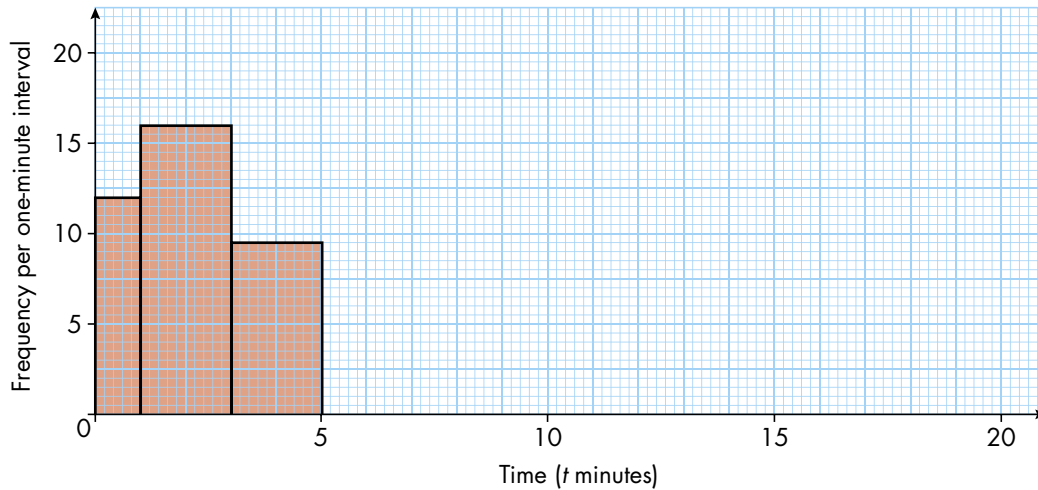
Times (t minutes)	Number of tracks
$t \leq 1$	0
$t \leq 2$	5
$t \leq 3$	
$t \leq 4$	
$t \leq 5$	
$t \leq 6$	
$t \leq 7$	200

- ii Draw the cumulative frequency curve showing the playing times of the CD tracks. Use a scale of 2 cm to 1 minute on the horizontal axis and 2 cm to 50 tracks on the vertical axis. [1]
 b Use the cumulative frequency curve to estimate the median playing time of a CD track [1]

- 2 A sample was taken of the telephone calls to a school switchboard. The lengths of the telephone calls are recorded, in minutes, in this table.

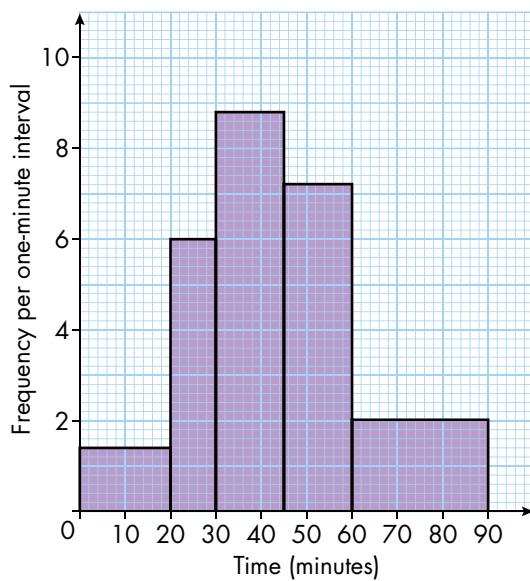
Time in minutes (t)	$0 < t \leq 1$	$1 < t \leq 3$	$3 < t \leq 5$	$5 < t \leq 10$	$10 < t \leq 20$
Number of calls	12	32	19	20	15

Copy and complete the histogram to show this information.



[3]

- 3 All the students in a small secondary school were asked how long, on average, they spent each night completing their homework. The results are shown in the histogram.

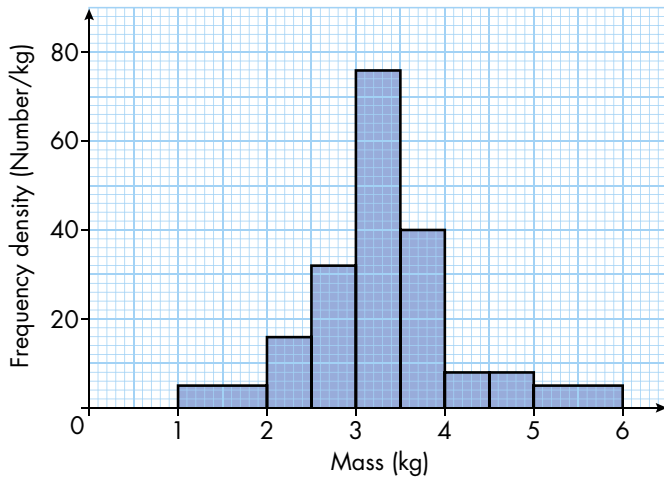


[3]

Copy and complete the following table of frequencies.

Time (minutes)	0–20	20–30	30–45	45–60	60–90
Frequency		60			

- 4 This histogram shows the masses of 100 babies born at a hospital.



- a How many babies had a mass between 2.5 kg and 4 kg? [3]
 b What was the modal group? [1]

Now try these exam questions

- Rearrange the formula $P = 5\sqrt{V}$ to make V the subject. [2]
- Given that $V = \frac{1}{3}\pi r^2 h$, express r in terms of V , h and π . [2]
- Rearrange each of the following to give d in terms of e .
 - $de = 5d + 3$ [3]
 - $\frac{3d - 7}{4 + 5d} = e$ [4]
- You are given that $f(x) = 5x - 4$
 - Find $f(0.5)$ [1]
 - Solve $f(x) = 0$ [1]
 - Express $f(1 + 2x)$ as simply as possible [2]

More exam practice

- Given that $f = \frac{uv}{u + 5}$, express v in terms of u and f . [3]
- The cost, C pence, of printing n party invitations is given by $C = 120 + 4n^2$. Find a formula for n in terms of C . [2]
- The surface area of an open cylinder is given by the formula $S = 2\pi r(r + 2h)$. Make h the subject of the formula. [3]
- $f(x) = 1 + 2x$ and $g(x) = 3 - 5x$
 - Find $f(3)$ and $g(3)$ [2]
 - Solve $f(2x) = g(x)$ [3]

16 Formulae 2

Here is an exam question ...

Use the formula $F = 2(C^2 + 15)$ to find an expression for C in terms of F . [3]

... and its solution

$$F = 2(C^2 + 15)$$

$$F = 2C^2 + 30$$

$$2C^2 = F - 30$$

$$C^2 = \frac{1}{2}(F - 30)$$

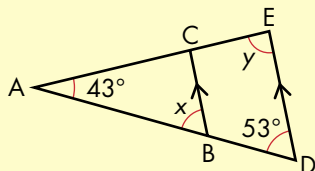
$$C = \sqrt{\frac{1}{2}(F - 30)}$$

1 Properties of shapes

Angles made with parallel lines

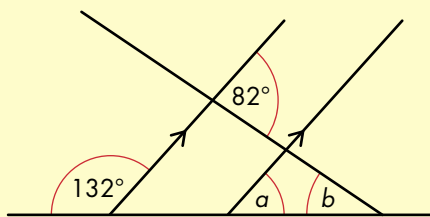
Try these exam questions

- 1 In the diagram, BC is parallel to DE.

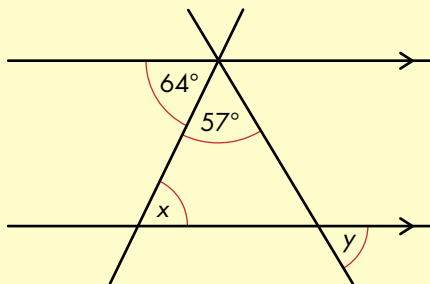


Find the size of x and y . Give reasons for your answers. [4]

- 2 Calculate the size of angles a and b . Give reasons for your answers. [4]



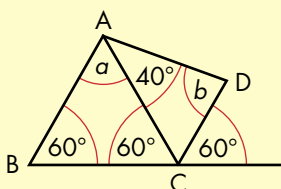
- 3 Calculate the size of x and y . Give reasons for your answers. [4]



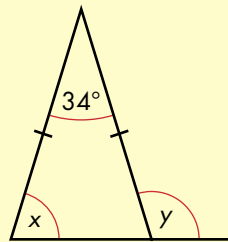
The angles in a triangle

Try these exam questions

- 1 a Find the size of angle a . [1]
 b What type of triangle is ABC? [1]
 c Find the size of angle b . [1]



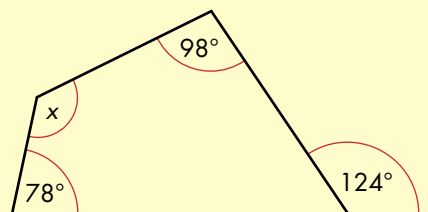
- 2 Find the size of x and y . Give reasons for your answers. [4]



The angles in a quadrilateral

Try this exam question

- 1 Calculate angle x . [4]



Special quadrilaterals

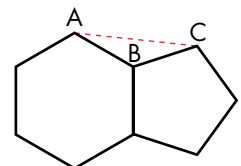
Try these exam questions

- 1 a Sketch a rhombus and mark everything that is equal. [2]
 b Draw in all the lines of symmetry. [1]
 c State the order of rotation symmetry. [1]
 2 A pentagon has four interior angles of 75° , 96° , 125° and 142° . Find the size of the fifth interior angle. [3]

The angles in a polygon

Here is an exam question ...

- a The sketch shows a regular pentagon and a regular hexagon with equal length sides.



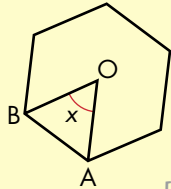
- i Work out the size of one interior angle of the pentagon and the hexagon. [2]
 ii Work out the sizes of the angles in triangle ABC. [2]

... and its solution

- a i** Exterior angles of a polygon add up to 360° .
 Exterior angle of pentagon = $\frac{360^\circ}{5} = 72^\circ$
 Interior angle = $180^\circ - 72^\circ = 108^\circ$
 Exterior angle of hexagon = $\frac{360^\circ}{6} = 60^\circ$
 Interior angle = $180^\circ - 60^\circ = 120^\circ$
- ii** Angles round a point add up to 360° .
 Angle ABC = $360^\circ - 120^\circ - 108^\circ = 132^\circ$
 Triangle ABC is isosceles.
 AB = BC
 Angles in a triangle add up to 180° .
 So angle BAC = angle BCA = $\frac{180^\circ - 132^\circ}{2}$
 $= \frac{48^\circ}{2} = 24^\circ$

Now try these exam questions

- 1** Here is a sketch of a regular pentagon, centre O.



- a** Work out x . [2]
b What type of triangle is OAB? [1]
- 2** The interior angle of a regular polygon is 168° . Find the number of sides of the polygon. [3]

2 Fractions, decimals and percentages

Comparing fractions

Try these exam questions

- 1 a** Put these fractions in order of size, smallest first.
 $\frac{3}{4}, \frac{7}{10}, \frac{3}{5}, \frac{5}{8}$ [2]
- b** Write these numbers in order, smallest first.
 66%, $\frac{3}{5}$, 0.62, 0.59, 55% [2]
- 2** Show that $\frac{4}{5}$ is bigger than $\frac{3}{4}$. [2]

- 3** Which of the following fractions are equal to $\frac{2}{3}$?

$$\frac{6}{10}, \frac{4}{6}, \frac{10}{15}, \frac{4}{9}, \frac{3}{2}$$

[2]

- 4** These are the lengths of four nails in inches.

$$1\frac{3}{5}, 1\frac{7}{16}, 1\frac{1}{4}, 1\frac{3}{8}$$

Put them in order, smallest first.

[2]

Adding and subtracting fractions and mixed numbers

Here is an exam question ...

Jane spent $\frac{1}{3}$ of her pay, gave her mother $\frac{2}{5}$ of her pay and saved the rest.

What fraction of her pay did Jane save? [3]

... and its solution

$$\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$$

$$\text{She saved } 1 - \frac{11}{15} = \frac{4}{15}$$

Now try these exam questions



- 1** Work out $1\frac{1}{4} + 2\frac{3}{5}$, giving your answer as simply as possible. [3]
- 2** $\frac{1}{4}$ of the number of cows in a field were Jerseys, $\frac{2}{3}$ were Friesians and the rest were Guernseys. What fraction were Guernseys? [2]
- 3** Work out $4\frac{3}{16} - 2\frac{1}{2}$. [3]
- 4** Two nails have length $1\frac{7}{16}$ and $1\frac{1}{4}$ inches. Find the difference between their lengths. [2]
- 5** Work out $\frac{2}{3} + \frac{4}{5}$, giving your answer as simply as possible. [2]

More exam practice

- 1** Work out $2\frac{3}{8} - 1\frac{1}{2}$, giving your answer as a fraction, as simply as possible. [3]
- 2** A piece of metal is $2\frac{1}{4}$ inches long. Stuart cuts off $\frac{7}{16}$ of an inch. How much is left? [3]
- 3** Work out $2\frac{1}{3} + 4\frac{1}{2}$, writing the answer as a fraction. [2]
- 4** Work out $2\frac{3}{8} - 1\frac{1}{2}$, giving your answer as a fraction in its lowest terms. [2]

Multiplying and dividing fractions and mixed numbers

Here is an exam question ...

Measurements used to be made in inches and fractions of an inch.

- a** Find the area of a rectangle measuring $2\frac{1}{2}$ inches by $5\frac{1}{4}$ inches. [2]
- b** Another rectangle has area $11\frac{1}{10}$ square inches. One side measures $1\frac{1}{2}$ inches. How long is the other side? [2]

... and its solution

- a** $2\frac{1}{2} \times 5\frac{1}{4} = \frac{5}{2} \times \frac{21}{4}$ [1]
 $= \frac{105}{8}$ or $13\frac{1}{8}$ [1]
 Square inches [1]
- b** $11\frac{1}{10} \div 1\frac{1}{2} = \frac{111}{10} \div \frac{3}{2}$ [1]
 $= \frac{111}{10} \times \frac{2}{3}$ [1]
 $= \frac{37}{5}$ or $7\frac{2}{5}$ inch [1]

Now try this exam question



- 1** Work out the following, giving your answers as simply as possible. [2]
- a** $\frac{3}{5} \times \frac{5}{6}$ [2]
- b** $\frac{2}{3} \div \frac{4}{5}$ [2]

More exam practice

- 1** Work out $1\frac{3}{5} \times 2\frac{2}{9}$. Give your answer as a fraction as simply as possible. [3]
- 2** $h = \frac{2xy}{x+y}$. Find h when $x = \frac{2}{5}$ and $y = \frac{2}{7}$. [6]
- 3** Work out these, giving your answers as fractions as simply as possible. [2]
- a** $\frac{3}{5} \times \frac{4}{9}$ [2]
- b** $\frac{3}{10} \div \frac{4}{15}$ [2]
- 4** Work out these, giving your answers as fractions in their lowest terms. [2]
- a** $2\frac{1}{3} \times 4\frac{1}{2}$ [2]
- b** $2\frac{1}{4} \div 1\frac{4}{5}$ [2]

Adding and subtracting decimals

Try these exam questions

- 1** $32 + 68$ [1]
- 2** $179 + 312$ [1]
- 3** $92 - 71$ [1]
- 4** $917 - 38$ [1]
- 5** $4321 - 2314$ [1]
- 6** $6.3 + 5.9$ [1]
- 7** $9.02 + 3.67$ [1]
- 8** $5.9 - 2.3$ [1]
- 9** $94.7 - 5.9$ [1]
- 10** $21.5 - 0.34$ [1]

Multiplying and dividing decimals

Try these exam questions

- 1** Seven bottles of cola cost £2.45. What do twelve bottles cost? [1]
- 2** James has a Saturday job. He earns £11.20 for 4 hours' work. How much will he get if he works for
- a** 3 hours? [1]
- b** 7 hours? [1]
- 3** 2 kilograms of cheese costs £6.70. How much does 5 kilograms cost? [1]
- 4** 5 apples cost £1.15. How much will it cost for
- a** 2 apples? [1]
- b** 8 apples? [1]
- 5** This is a recipe for cheesecake for eight people.
- | | |
|----------------|-------|
| Biscuit crumbs | 160 g |
| Butter | 40 g |
| Cream cheese | 640 g |
| Caster sugar | 70 g |
- a** How much butter is needed to make a cheesecake for five people? [1]
- b** How much cream cheese is needed to make a cheesecake for twelve people? [1]

More exam practice

- 1** Work out $15.7 - 3.9 \times 2$. [2]
- 2** On their holidays, Sue and Pam drove 178 miles on the first day and 274 miles on the second day.
- a** How far did they drive in those two days? [2]
- b** How much further did they drive on the second day? [2]

- 3 Serina goes to a garden centre.
- She buys two bags of fertilizer at £2.27 each and a trowel at £4.56. Work out how much change she gets from a £20 note. [3]
 - She later buys 18 packets of seeds at 82p a packet. Work out the total cost of the 18 packets of seeds. Give the answer in pounds. [3]

Percentage increase and decrease

Try these exam questions

- A calculator was sold for £6.95 plus VAT when VAT was 17.5%.
What was the selling price of the calculator including VAT?
Give the answer to the nearest penny. [3 + 1]
- All clothes in a sale were reduced by 15%.
Mark bought a coat in the sale that was usually priced at £80.
What was its price in the sale? [3]

More exam practice

- A bath normally priced at £750 is offered with a discount of 10%. What is the new price of the bath? [3]
- In a sale, all the prices were reduced by 20%.
A jumper was originally priced at £45. What was the sale price? [3]
- A low-sugar jam claims to have 42% less sugar.
A normal jam contains 260 g of sugar. How much sugar does the low-sugar jam contain? [3]
- Stephen negotiated a 5% reduction in his rent.
It was originally £140 a week. What was it after the reduction? [3]

3 Mental methods

Mental strategies

Try these exam questions

- Work out these.
 - 4.3×2.6 [3]
 - $16.33 \div 2.3$ [3]
 - $471.5 \div 100$ [1]
- Write down the answers to these questions.
 - 5^2 [1]
 - 11^2 [1]
 - $\sqrt{16}$ [1]
 - $\sqrt{81}$ [1]
 - 4^3 [1]
- Work out these.
 - 3.2×200 [1]
 - $804 \div 40$ [1]
 - $20\,000 \div 5\,000$ [1]

Rounding to a given number of significant figures

Try these exam questions

- Write down the value of 14.89 correct to 1 significant figure. [1]
- Sam got the answer 3246.963 34 to a problem on his calculator.
Write 3246.963 34 correct to
 - the nearest whole number. [1]
 - 1 significant figure. [1]
 - 1 decimal place. [1]

More exam practice

- The average weight of a member of England's rugby scrum was 128.825 kg.
Round this to
 - the nearest whole number. [1]
 - 1 decimal place. [1]
- Write 572 to the nearest 100. [1]
 - Write 2449 to the nearest 1000. [1]

Deriving unknown facts from those you know

Try these exam questions

- 1 Mrs Brown took 47 students to London. The train tickets cost £20.25 each. She worked out the cost as £770.75.
Do an estimate to show that this must be the wrong answer. [2]
- 2 Graham sent 78 Christmas cards. He put a second class stamp, which cost 32p each, on every envelope. Estimate how much it cost him for the stamps. [2]
- 3 During his summer holidays, Eric drove 2778 miles and used 287 litres of petrol. To find how many miles the car travelled per litre, he worked out $\frac{2778}{287}$.
Write down an estimate of this calculation, showing your approximations. [2]

More exam practice

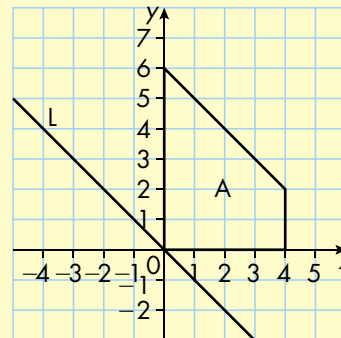
- 1 Francis has £45 to spend at the garden centre. He wants to buy a bird table costing £23.85 and six bags of birdseed costing £2.95 each. Show how he can work out in his head that £45 will be enough. Do not work out the exact amount. [2]
- 2 Estimate the value of 74.2×179 . [2]
- 3 Joe was earning £20 840 a year. He was given a 5% rise. He worked out his new salary as £31 260. Why is it obvious his answer is wrong? [2]
- 4 Pat calculated $\frac{5857}{62 \times 20.3}$ as 1917.70. Make an estimate to show this is wrong. [2]

4 Transformations

Reflections

Try this exam question

- 1 The diagram shows the shape A and the line L.



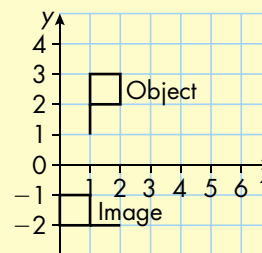
Draw the image of shape A after reflection in the line L. Note that you will need x - and y -axes from -6 to 6 .

[4]

Rotations

Try this exam question

- 1 Describe this transformation.



Translations

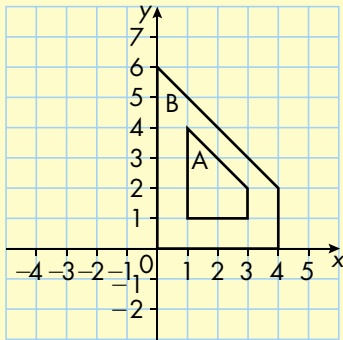
Try this exam question

- 1 Draw axes from 0 to 6 for x and y . Plot the points (3, 1), (3, 2), (3, 3) and (4, 2) and join to form a flag, A.
Translate flag A by $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Label the image B.

Enlargements

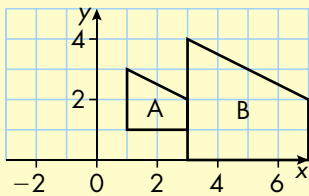
Try these exam questions

- 1 The diagram shows the shapes A and B.



Shape B is an enlargement of shape A. For this enlargement, find

- a the scale factor. [1]
 b the coordinates of the centre of enlargement. [1]
- 2 Find the centre and scale factor of the enlargement that maps shape A on to shape B. [3]



Combining transformations

Here is an exam question ...

Shape A is translated by $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$. Its image is B.

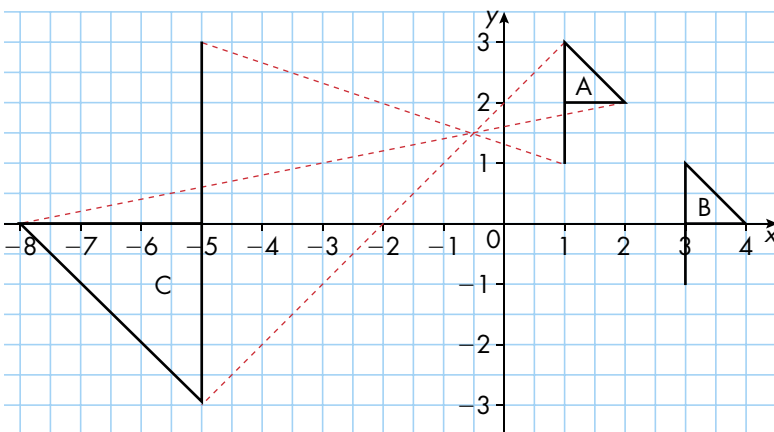
Shape B is enlarged with centre $(1, 0)$ and scale factor -3 .
 Its image is C.

Describe the transformation which maps C on to A.

[7]

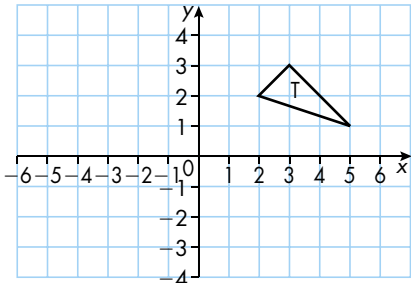
... and its solution

The transformation which maps C on to A is an enlargement with centre $(-0.5, 1.5)$ and scale factor $-\frac{1}{3}$.



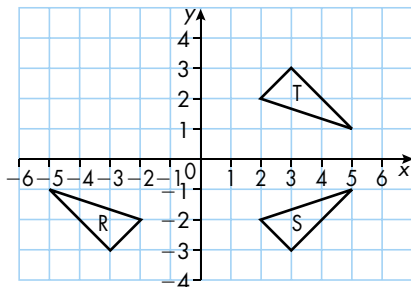
Here is another exam question ...

- Triangle T is rotated 180° clockwise about the point $(0, 0)$. Its image is triangle R. Draw and label triangle R. [2]
- Triangle R is reflected in the y -axis. Its image is triangle S. Draw and label triangle S. [1]
- Describe the single transformation which would map triangle T on to triangle S. [3]



... and its solution

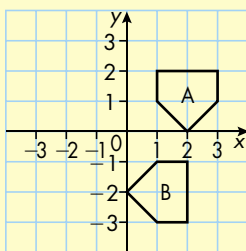
a and b



- Reflection in the x -axis. [1]

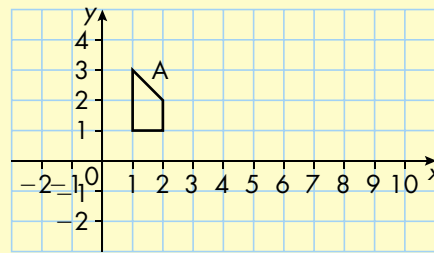
Now try these exam questions

- The diagram shows shapes A and B.

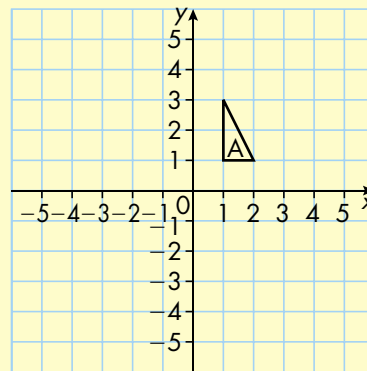


- Describe fully the single transformation that maps shape A on to shape B. [2]
- Draw the image of shape A after a reflection in the y -axis. Label the image C. [2]
- Draw the shape A after an enlargement with centre $(0, 0)$ and scale factor 3. Label the image D. Note that you will need an x -axis from -5 to 10 and a y -axis from -5 to 8 . [3]

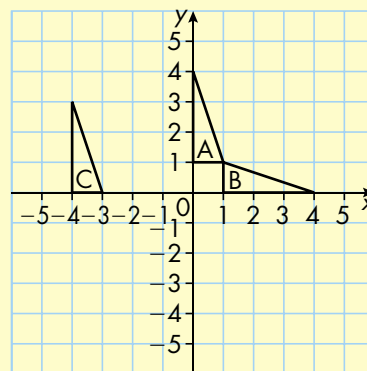
- Reflect shape A in the y -axis. Label the image B. [1]
 - Reflect shape B in the line $x = 3$. Label the image C. [1]
 - Write the column vector for the translation which would map A on to C. [2]



- Reflect triangle A in the line $x = 3$. Label the image B. [2]
 - Translate triangle A by $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$. Label the image C. [2]



- Describe fully the single transformation that maps triangle A on to triangle B. [2]
 - Rotate triangle C through 90° clockwise about $(-4, -1)$. Label the image D. [2]



5 Straight-line graphs

Drawing straight-line graphs and harder straight-line graphs

Try these exam questions

Draw the following straight-line graphs.

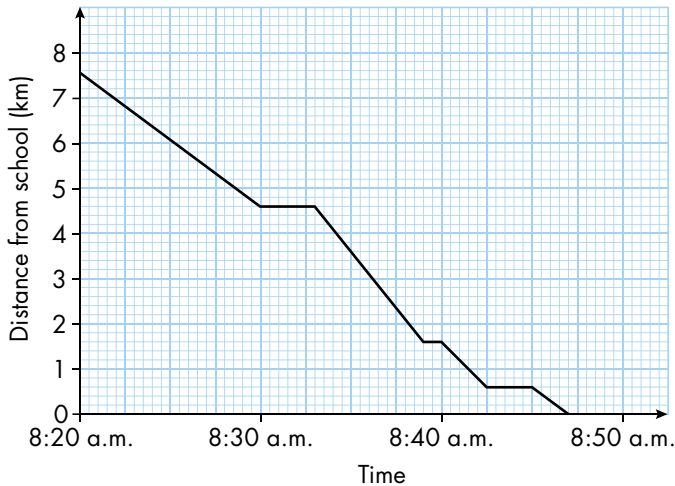
1 $y = 6 - 2x$

2 $4x + 7y = 28$

Distance–time graphs

Here is an exam question ...

Tom leaves home at 8.20 a.m. and goes to school on a moped.
The graph shows his distance from the school in kilometres.



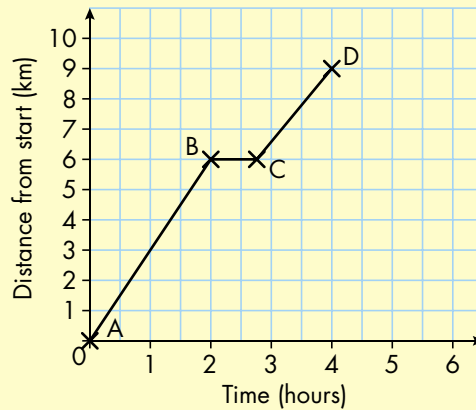
- a How far does Tom live from school? [1]
 b Write down the time that Tom arrives at the school. [1]
 c Tom stopped three times on the journey.
 For how many minutes was he at the last stop? [1]
 d Calculate his speed in km/h between 8.20 a.m. and 8.30 a.m. [2]

... and its solution

- a 7.6 km
 b 8.47 a.m.
 c 2.5 minutes
 d Distance = $7.6 - 4.6 = 3$ km
 Time = 10 mins
 Speed = $\frac{3}{10} \times 60 = 18$ km/h

Now try this exam question

- 1 Jim went out walking.
In the diagram ABCD represents his walk.

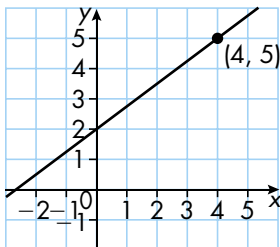


- a How far had Jim walked after $1\frac{1}{2}$ hours? [1]
 b What does the part of the graph BC represent? [1]
 c After walking 9 km, Jim turned round and walked straight back to his starting place without stopping. It took him 2 hours to get back. Draw a line on a copy of the grid to show this. [2]
 d Work out his average speed on the return journey. [2]

Finding the gradient and equation of a straight-line graph

Here is an exam question ...

Find the gradient and the equation of the straight line in the diagram. [3]

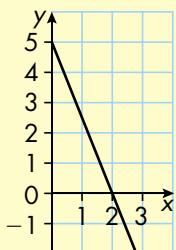


... and its solution

Gradient = $\frac{3}{4}$
 Equation is $y = \frac{3}{4}x + 2$

Now try these exam questions

- 1 Work out the gradient of this line. [2]



- 2 a Write down the gradient and y -intercept of the line with equation $y = 4 - 2x$. [2]
 b Write down the equation of the line parallel to $y = 4 - 2x$ and passing through the point $(0, -1)$. [2]
 3 Find the gradients and y -intercepts of these lines. [3]
 a $y = \frac{1}{2}x + 3$ [2]
 b $2y = 5 - 3x$ [2]
 c $4x + 3y = 12$ [2]

Exploring gradients

Try this exam question

- 1 Find the equations of two straight lines which pass through the point $(1, 1)$, one parallel to $x + y = 1$ and the other perpendicular to $x + y = 1$. [4]

6 Indices, decimals and surds

The rules of indices and Using the rules of indices with numbers and letters

Here is an exam question ...

- a Simplify these.
- i $\frac{(pq^3)^2}{p^3q}$ ii $(p^8)^{-\frac{3}{4}}$ [5]

... and its solution

- a i $\frac{p^2q^6}{p^3q} = p^{-1}q^5 = \frac{q^5}{p}$ For p : $2 \times 1 - 3 = -1$
For q : $3 \times 2 - 1 = 5$
- ii $p^{-6} = \frac{1}{p^6}$ $8 \times -\frac{3}{4} = -6$

Here is another exam question ...

Simplify $\frac{2a^4 \times 4a^2}{a^3}$. [2]

... and its solution

$$\frac{8a^6}{a^3} = 8a^3$$

Now try these exam questions

- 1 Simplify these.
- a $3a^2b \times 4ab^3$ [2]
- b $\frac{a^3b^5}{a^2b^3}$ [2]
- 2 Simplify $\frac{m^3n^3 \times m^4n^2}{m^5n}$ [3]
- 3 Simplify $x^5 \times x^2$ [1]
- 4 Simplify $\frac{p^2 \times p^6}{p^3}$ [2]

Terminating and recurring decimals

Try this exam question

- 1 a Express $0.\dot{1}4$ as a fraction in its lowest terms. [2]
- b Without working them out, state whether these fractions are equal to terminating or recurring decimals. Explain your answers.
- i $\frac{4}{7}$ [1]
- ii $\frac{3}{40}$ [1]
- iii $\frac{5}{6}$ [1]
- iv $\frac{4}{9}$ [1]

Surds

Try these exam questions

- 1 a Express the following in the form $p\sqrt{q}$ where p and q are integers and q is as small as possible. For example $\sqrt{8} = 2\sqrt{2}$.
- i $\sqrt{72}$ [2]
- ii $\sqrt{20} \times \sqrt{15}$ [2]
- iii $\frac{\sqrt{50} \times \sqrt{27}}{\sqrt{18}}$ [2]
- b Given that $(5 + \sqrt{7})^2 = a + b\sqrt{7}$, find the values of a and b . [2]
- 2 It is given that $p = 3 + \sqrt{5}$ and $q = 2 - 3\sqrt{5}$. Simplify the following expressions, giving each answer in the form $a + b\sqrt{5}$, where a and b are integers.
- a $p + q$ [2]
- b p^2 [2]
- c pq [2]
- 3 Given that $p = (2 + \sqrt{7})$ and $q = (2 - \sqrt{7})$, simplify the following expressions, showing all your working.
- a $p + q$ [2]
- b $p - q$ [2]
- c pq [2]

7 Inequalities

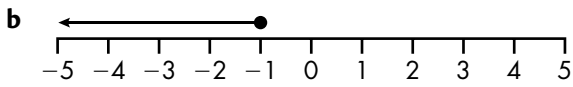
Solving inequalities with one unknown

Here is an exam question ...

- a Solve $3x + 4 \leq 1$. [2]
- b Show your solution to part a) on a number line. [1]

... and its solution

a $3x \leq 1 - 4$
 $3x \leq -3$
 $x \leq \frac{-3}{3}$
 $x \leq -1$

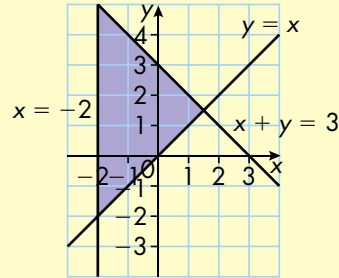


Now try these exam questions

- List the integers for which $-2 \leq x < 5$. [2]
- Solve these inequalities and show the solutions on number lines.
 - $8x + 5 > 25$ [2 + 1]
 - $2x + 17 > 4x + 6$ [2 + 1]

Now try these exam questions

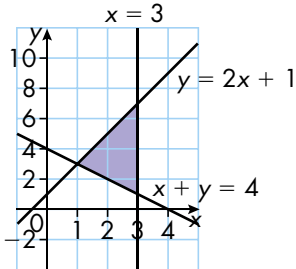
- Find the region satisfied by the inequalities $x + y \leq 5$, $y \leq 2x - 1$, $y \geq 0$. [5]
- Look at the graph. Write down the three inequalities which are satisfied in the shaded region. [3]



Solving inequalities with two unknowns

Here is an exam question ...

Write down the three inequalities that are satisfied in the shaded region.



... and its solution

Choose a point in the region, say $x = 2$, $y = 3$.
 $y = 2x + 1$
 $3 \leq 2 \times 2 + 1$ so $y \leq 2x + 1$
 $x = 2$, $2 \leq 3$ so $x \leq 3$
 $x + y = 4$, $2 + 3 \geq 4$ so $x + y \geq 4$
 So inequalities are
 $y \leq 2x + 1$, $x \leq 3$, $x + y \geq 4$

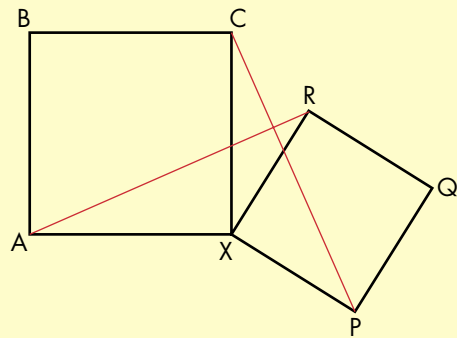
8 Congruency

Congruent triangles

Try this exam question

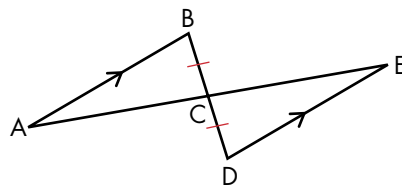
- The diagram shows two squares, ABCX and PQRX. Prove that angles PCX and RAX are equal. [4]

[3]

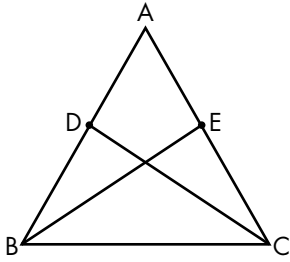


More exam practice

- In the diagram, AB is parallel to DE and $BC = CD$. State, with reasons, which triangle is congruent to triangle ABC. [3]



- 2 ABC is an isosceles triangle where $AB = AC$, and D and E are the midpoints of AB and AC. Use congruent triangles to prove that $EB = DC$. [4]



9 Simultaneous equations

Solving simultaneous equations graphically

Try this exam question

- 1 Solve these simultaneous equations graphically.
 $y = 3 - x$
 $y = 3x - 2$ [4]

More exam practice

- 1 a Draw these graphs on the same axes.
 i $y = 3x - 1$ [3]
 ii $2x + 3y = 12$ [3]
 b Write down the coordinates of the point where the two graphs cross. [1]

Solving simultaneous equations algebraically

Try this exam question

- 1 Solve algebraically these pairs of simultaneous equations.
 a $5x + 4y = 13$
 $3x + 8y = 5$ [3]
 b $4x + 3y = 5$
 $2x + y = 1$ [3]
 c $y = 3x - 7$
 $2x + 3y = 1$ [3]

Solving harder simultaneous equations algebraically

Try this exam question

- 1 Solve algebraically these pairs of simultaneous equations.
 a $2x - 3y = 9$
 $5x + 2y = -25$ [4]
 b $3x - 5y = 11$
 $4x - 3y = 11$ [4]

10 Vectors

The definition of a vector

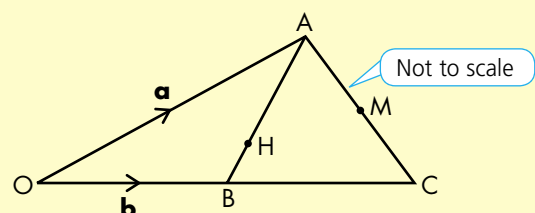
Try this exam question

- 1 [3]
 a Work out these.
 i \vec{AB}
 ii \vec{BC}
 iii \vec{CA} [3]
 b What do you notice about $\vec{AB} + \vec{BC} + \vec{CA}$? [1]
 c Calculate the length of \vec{BC} , showing your working. [3]

Multiplying a vector by a scalar

Try this exam question

- 1 In the diagram, $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$. M is the midpoint of AC and B is the midpoint of OC.



- a** Write down the following vectors in terms of **a** and **b**, simplifying your answers where possible.
- i** \vec{OC} [1] **ii** \vec{BA} [1] **iii** \vec{OM} [1]
- b** H is a point on BA and $\vec{BH} = \frac{1}{3}\vec{BA}$.
Find the vector \vec{OH} in terms of **a** and **b**. [3]
- c** What do your results tell you about the points O, H and M? [2]

Addition and subtraction of vectors

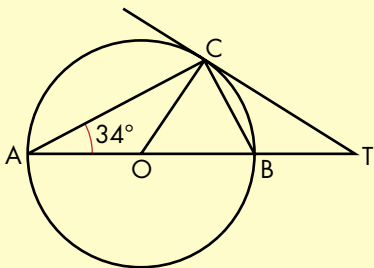
Try this exam question

- 1** A and C have coordinates $(-2, 1)$ and $(2, -2)$ respectively. $\vec{AB} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$. M is the midpoint of AB and N is the midpoint of AC.
- a** Write down the coordinates of B, M and N. [3]
- b** Calculate the vectors \vec{BC} and \vec{MN} . [2]
- c** State the relationship between BC and MN. [2]

11 Circle theorems

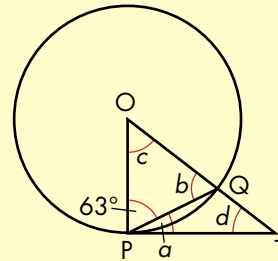
Try these exam questions

- 1** The diagram shows a circle, centre O. The diameter AB is produced to T and TC is a tangent to the circle. Calculate these angles.

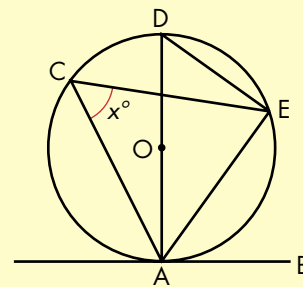


- a** $\angle OCB$
b $\angle CBT$
c $\angle CTA$ [6]

- 2** In the diagram, O is the centre of the circle, PT is the tangent to the circle at P and Q is the point where the line OT cuts the circle. Calculate the sizes of the angles marked *a*, *b*, *c* and *d*. [4]



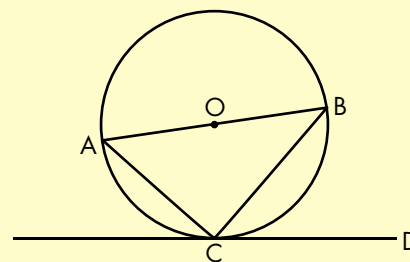
- 3** In the diagram, O is the centre of the circle, AD is a diameter and AB is a tangent. Angle ACE = x° .



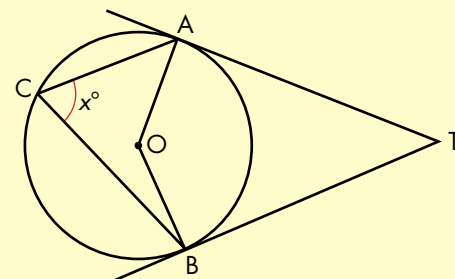
Find, in terms of *x*, the size of these angles.

- a** $\angle ADE$ [1]
b $\angle DAE$ [1]
c $\angle EAB$ [1]
d $\angle AOE$ [1]
- 4 a** AB is a diameter of the circle centre O. CD is a tangent touching the circle at C. Angle BCD = 53° . Write down the size of each of the following angles, giving your reasons.

- i** $\angle ACB$ **ii** $\angle BAC$



- b** TA and TB are tangents to the circle centre O. Angle ACB = x° . Find the size of angle ATB in terms of *x*. [8]



12 Scatter diagrams and time series

Scatter diagrams

Try these exam questions

- 1 An orchard contains nine young apple trees. The table shows the height of each tree and the number of apples on each.

Height (m)	Number of apples
1.5	12
1.9	15
1.6	20
2.2	17
2.1	20
1.3	8
2.6	26
2.1	22
1.4	10

- a Draw a scatter diagram to illustrate this information. Use a scale of 2 cm to 1 m on the horizontal axis and 2 cm to 10 apples on the vertical axis. [2]
- b Comment briefly on the relationship between the height of the trees and the number of apples on the trees. [1]
- c Add a line of best fit to your scatter diagram. [1]
- d Explain why it is not reasonable to use this line to estimate the number of apples on a tree of similar type but of height 4 m. [1]
- 2 Bob thinks there is a relationship between arm length and the length of the first finger of the writing hand. He measured these lengths for 14 students in his class. The measurements, in cm, are given in the table below.

Arm	Finger
56	6
65.5	6.5
73.5	7.5
64	7
64	6
81	8
57	6.5
60	6.5
69	7
53	6
65	7
72.5	7.5
57.5	7
74.5	7.5

- a Draw a scatter graph to show this data. [3]
- b What does the graph show about any connection between the two lengths? [1]
- c Bob knows the arm length for a student is 68 cm. What would you expect the finger length to be? [2]

Time series

Try this exam question

- 1 The table shows a company's quarterly sales of umbrellas in the years 2007 to 2010. The figures are in thousands of pounds.

	1st quarter	2nd quarter	3rd quarter	4th quarter
2007	153	120	62	133
2008	131	105	71	107
2009	114	110	57	96
2010	109	92	46	81

- a Plot these figures on a graph. Use a scale of 1 cm to each quarter on the horizontal axis and 2 cm to 20 thousand pounds on the vertical axis. [3]

- b** The four-quarter moving averages are a , b , 107.5, 110, 104.5, 99.25, 100.5, 97, 94.25, 93, 88.5, 85.75 and 82.
Calculate the values of a and b , the first two moving averages. [2]
- c** Plot all the moving averages on your graph. [2]
- d** Comment on the general trend and the quarterly variation. [2]
- e** Draw a trend line for the points representing the moving averages. [1]
- f** Use your trend line to predict the next moving average (2nd quarter of 2010 to the 1st quarter of 2011). [1]
- g** Use the value you found in part **f** to predict the umbrella sales for the first quarter of 2011. [2]

1 Algebraic manipulation

Here is an exam question ...

Expand $(2q - 3)(q + 5)$.

[3]

... and its solution

x	2q	-3
q	$2q^2$	$-3q$
5	$10q$	-15

Answer:

$$2q^2 + 7q - 15$$

Now try these exam questions

1 Expand and simplify these.

a $(x + 3)(x + 7)$ [2]

b $(y + 4)(y - 3)$ [2]

c $(p - 2)(p - 6)$ [2]

2 Expand and simplify these.

a $(3 - x)^2$ [2]

b $(3x - 2)(x + 4)$ [2]

More exam practice

1 Simplify this expression.

$$(2e - f)(e + 3f)$$
 [2]

2 Simplify this expression.

$$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$$
 [2]

3 Multiply out and simplify these.

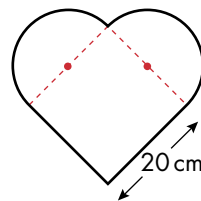
a $(2x + y)^2$ [2]

b $(2y - 1)(y - 3)$ [2]

2 Perimeter, area, volume and 2-D representation

Here is an exam question ...

A heart shape is made from a square and two semi-circles.



Find the area and perimeter of the heart shape. [3]

... and its solution

Shape = square of side 20 cm + one whole circle of radius 10 cm

$$\begin{aligned} \text{Area of shape} &= 20 \times 20 + \pi \times 10^2 \\ &= 714.2 \text{ cm}^2 \text{ (to 1 d.p.)} \end{aligned}$$

Perimeter of shape = two semicircles + two sides of the square

$$= \text{circumference of whole circle} + 40 \text{ cm}$$

$$= \pi \times 20 + 40$$

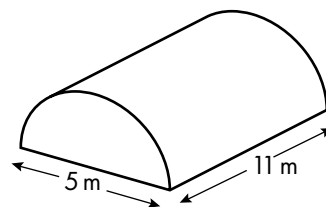
$$= 102.8 \text{ cm (to 1 d.p.)}$$

Here is another exam question ...

Find the volume of this greenhouse.

The ends are semicircles.

[3]



... and its solution

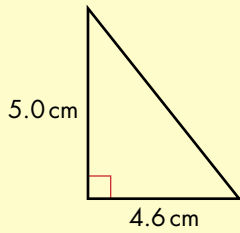
$$\begin{aligned} \text{Area of end} &= \frac{1}{2} \times \pi r^2 \\ &= \frac{1}{2} \times \pi \times 2.5^2 \end{aligned}$$

Volume = area of end \times length

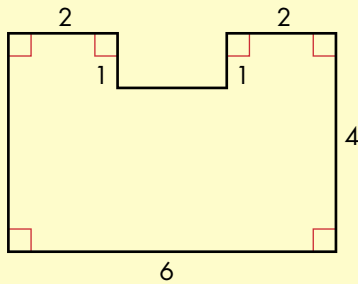
$$\begin{aligned} &= \left(\frac{1}{2} \times \pi \times 2.5^2\right) \times 11 \\ &= 108 \text{ m}^3 \text{ (to 3 s.f.)} \end{aligned}$$

Now try these exam questions

- 1 Find the area of this triangle.

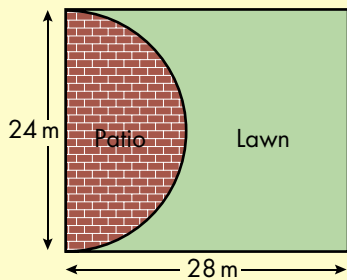


- 2 Mr Chan has drawn this plan of his lounge floor.



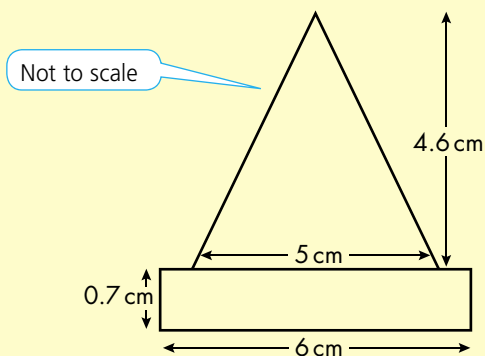
What is the perimeter and area of his lounge floor?
All lengths are in metres. [4]

- 3 Work out the area of the lawn in this diagram. [4]

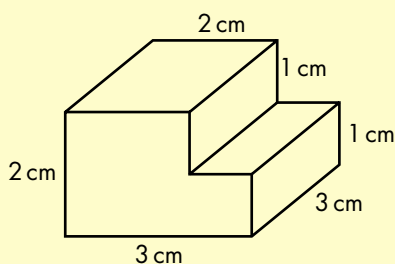


- 4 The circumference of a circle is 26 cm.
Calculate the radius of this circle. [2]

- 5 Find the total area of this shape. [4]

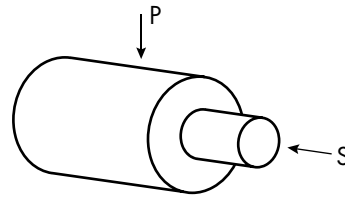


- 6 Calculate the volume of this prism. [3]

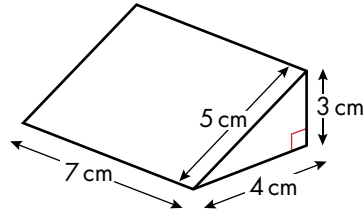


More exam practice

- 1 Sketch the plan (P) and side elevation (S) of this shape. [3]

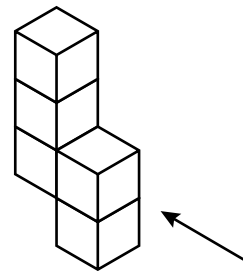


- 2 This is a triangular prism.

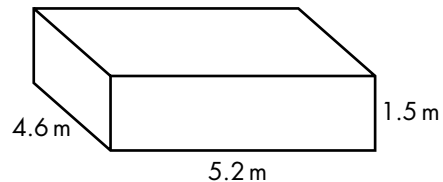


- a Find its volume.
b Find its surface area. [6]

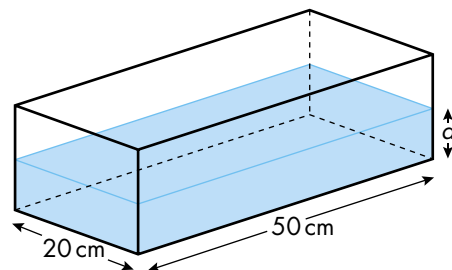
- 3 This shape is made from five cubes.
Draw the plan view of the shape and the view from the direction of the arrow. [4]



- 4 Calculate the volume of this cuboid. [2]

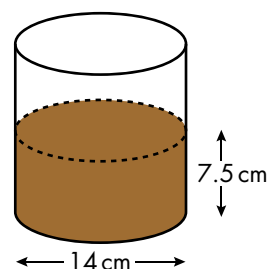


- 5 The volume of water in this fish tank is $10\,000\text{ cm}^3$.
All the sides and base of the tank are rectangles.



- Calculate the depth of water in the tank. [3]

- 6 Find the volume of coffee in this cylindrical tin. [3]



3 Trial and improvement

Here is an exam question ...

A solution of the equation $x^3 + 4x^2 = 8$ lies between -3 and -3.5 . Find this solution by trial and improvement. Give your answer correct to 2 decimal places. [4]

... and its solution

$$x = -3 \quad x^3 + 4x^2 = -27 + 36 = 9 \quad \text{Too big}$$

$$x = -3.5 \quad x^3 + 4x^2 = 6.125 \quad \text{Too small}$$

$$x = -3.3 \quad x^3 + 4x^2 = 7.623 \quad \text{Too small}$$

Answer lies between -3.3 and -3

$$x = -3.2 \quad x^3 + 4x^2 = 8.192 \quad \text{Too big}$$

Answer lies between -3.3 and -3.2

$$x = -3.25 \quad x^3 + 4x^2 = 7.921875 \quad \text{Too small}$$

Answer lies between -3.25 and -3.2

This solution keeps several decimal places as a check for you. There is no need to write them all down. e.g. for $x = -3.23$ $x^3 + 4x^2 = 8.03$ is enough.

$$x = -3.23 \quad x^3 + 4x^2 = 8.033333 \quad \text{Too big}$$

Answer lies between -3.23 and -3.25

$$x = -3.24 \quad x^3 + 4x^2 = 7.978176 \quad \text{Too small}$$

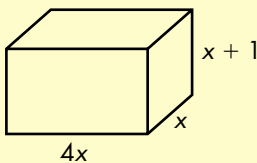
Answer lies between -3.23 and -3.24

$$x = -3.235 \quad x^3 + 4x^2 = 8.00589 \quad \text{Too big}$$

So the answer is between -3.235 and -3.24 and to 2 d.p. the answer is $x = -3.24$.

Now try these exam questions

- 1 The volume of this cuboid is 200 cm^3 .

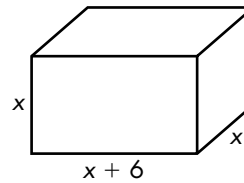


- a Explain why $x^3 + x^2 = 50$. [2]
 b Find the solution of $x^3 + x^2 = 50$ that lies between 3 and 4. Give the answer correct to 3 significant figures. You must show your trials. [4]

- 2 a Show that the equation $x^3 - 8x + 5 = 0$ has a solution between $x = 2$ and $x = 3$. [2]
 b Use trial and improvement to find this solution correct to 1 decimal place. Show all your trials and their outcomes. [4]

More exam practice

- 1 The equation $x^3 - 15x + 3 = 0$ has a solution between 3 and 4. Use trial and improvement to find this solution. Give your answer to 1 decimal place. Show clearly the outcomes of your trials. [4]
 2 Use trial and improvement to find the solution of $x^3 - 3x = 15$ between 2 and 3. Give your answer to 2 decimal places. Show clearly the outcomes of your trials. [4]
 3 Use trial and improvement to calculate, correct to 2 decimal places, the solution of the equation $x^3 - 5x - 2 = 0$ which lies between $x = 2$ and $x = 3$. Show all your trials and their outcomes. [4]
 4 The volume, $V \text{ cm}^3$, of this cuboid is given by $V = x^3 + 6x^2$.



- a Complete this table for values of x from 1 to 6.

x	1	2	3	4	5	6
V						

[2]

- b Use trial and improvement to find the dimensions of the cuboid if its volume is 200 cm^3 . Give the answer correct to 1 decimal place. Show all your trials. [4]

4 Probability 1

Here is an exam question ...

The cards used in a child's game have either a square, a triangle, a circle or a star printed on them.

The table shows the probabilities of getting the shapes.

Outcome	Square	Triangle	Circle	Star
Probability	0.2	0.35		0.3

- a Find the probability of getting a circle. [2]
The cards are either red or blue. There are three times as many blue cards as red cards.
- b What is the probability that the card drawn is red? [2]

... and its solution

- a All the probabilities must add up to 1.
 $0.2 + 0.35 + 0.3 = 0.85$
Therefore the probability getting a circle is 0.15.
- b 3 'parts' of the total are blue so 1 'part' is red.
The total number of 'parts' is 4.
So the probability that the card is red is $\frac{1}{4}$.

Now try these exam questions

- 1 Ahmed is counting vehicles passing a junction between 8.00 a.m. and 8.30 a.m.

Vehicle	Frequency
Car	72
Motorcycle	15
Lorry	28
Van	33
Bus	12

- a Use these data to find the probability that the next vehicle to pass the junction.
- is a car. [2]
 - is a bus. [1]
 - has more than two wheels. [2]
- Give your answers as fractions in their lowest terms.
- b Will this give reliable results for vehicles passing the junction at 11.00 p.m.? Explain your answer. [1]
- 2 There are only red, green and blue counters in a bag.
- a Copy and complete the table to show the probability of choosing at random a blue counter from the bag.
- | Colour | Red | Green | Blue |
|-------------|-----|-------|------|
| Probability | 0.2 | 0.45 | |
- b A counter is chosen from the bag, its colour is noted and it is then replaced. If this is repeated 200 times, how many counters would you expect to be green? [2]

More exam practice

- 1 Coloured sweets are packed in bags of 20. There are five different colours. The probabilities for four are given in the table.

Colour	Probability
Orange	0.05
White	
Yellow	0.2
Green	0.25
Red	0.35

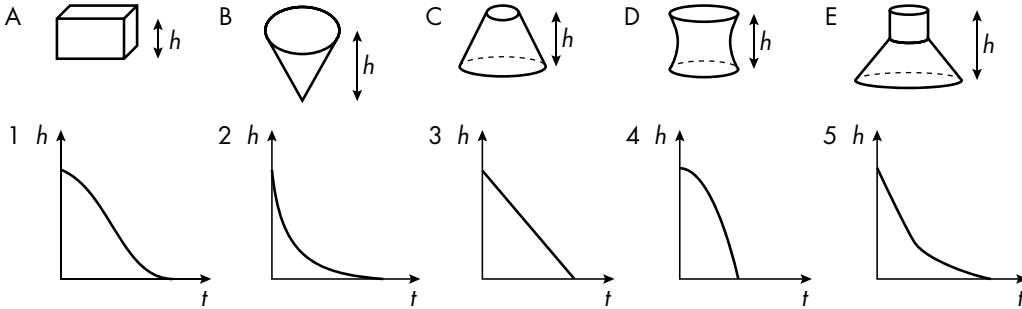
- a Find the probability of picking a white sweet. [2]
- b Find the probability of not picking a green sweet. [1]
- c How many sweets of each colour would you expect to find in a bag? [3]
- 2 A certain type of moth on a tropical island has either two spots, three spots, or four spots on its wings. The probability that a moth has two spots is 0.3. In a survey conducted by biologists, 1000 moths were examined and 420 moths with three spots were found. What is the probability of a moth, caught at random, having four spots? [3]
- 3 A fair coin shows heads or tails. A fair spinner is numbered 1, 2, 3, 4, 5.
- a Show all the possible outcomes when the coin is tossed and the spinner spun together. [2]
- b What is the probability that the outcome is heads and 5? [1]

5 Graphs 1

Real-life graphs

Here is an exam question ...

All these containers are full of liquid. The liquid runs out of each at a constant rate. Match each graph to the correct container.



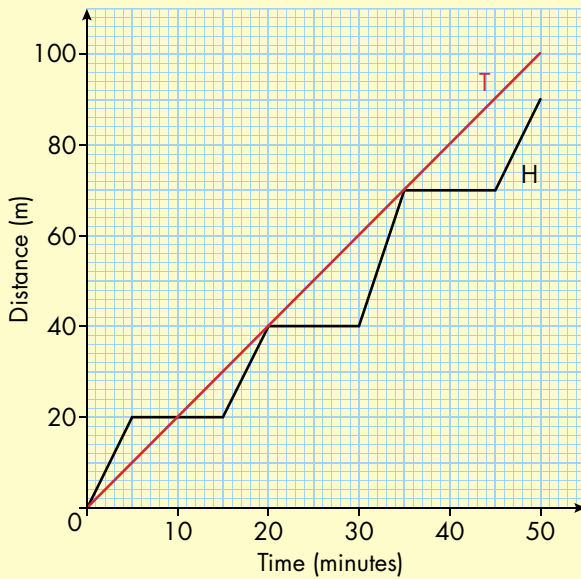
[3]

... and its solution

A ↔ 3 B ↔ 4 C ↔ 2 D ↔ 1 E ↔ 5

Now try this exam question

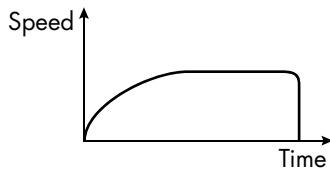
The graph shows the progress of the tortoise (T) and the hare (H).



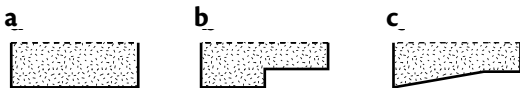
- a** Describe the movement of the tortoise. [2]
b i Describe the movements of the hare. [2]
ii What was his fastest speed? [2]

More exam practice

- 1 Steve travelled from home to school by walking to a bus stop and then catching a school bus.
- a Use the information below to construct a distance–time graph for Steve’s journey. Steve left home at 08.00. He walked at 6 km/h for 10 minutes. He then waited for 5 minutes before catching the bus. The bus took him a further 8 km to school at a steady speed of 32 km/h. [3]
- b How far was Steve from home at 08.20? [1]
- 2 The graph below describes a real-life situation. Describe a possible situation that is occurring. [2]



- 3 The diagrams show the cross-sections of three swimming pools. Water is pumped into all three at a constant rate. Sketch graphs of depth against time for each. [3]



Quadratic graphs

Here is an exam question ...

- a Copy and complete the table for $y = 2x^2 - 3x$ [2]

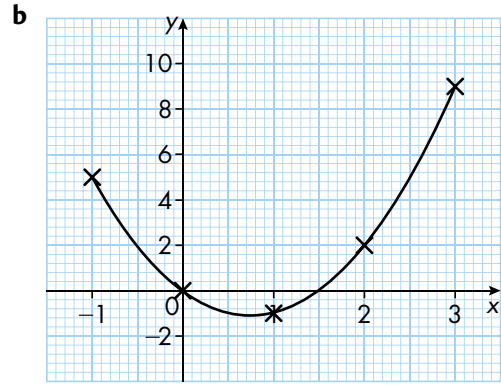
x	-1	0	1	2	3
y			-1	2	

- b Draw the graph of $y = 2x^2 - 3x$. [2]
 c Use your graph to solve $2x^2 - 3x = 0$. [2]

... and its solution

a

x	-1	0	1	2	3
y	5	0	-1	2	9



- c Solutions where the curve crosses the x -axis: $x = 0$ or $x = 1.5$

Now try these exam questions

- 1 a Copy and complete the table of values and draw the graph of $y = x^2 - 2x + 1$ for values of x from -1 to 3 . [4]

x	-1	0	1	2	3
y		1			4

- b Use your graph to find the values of x when $y = 3$. [2]

- 2 a Copy and complete the table for $y = 4x - x^2$ and draw the graph. [4]

x	-1	0	1	2	3	4	5
y			3			0	

- b Use your graph to find

- i the value of x when $4x - x^2$ is as large as possible. [2]

- ii between which values of x the value of $4x - x^2 - 2$ is larger than 0. [2]

More exam practice

- 1 a Copy and complete the table for $y = x^2 - 2$. [2]

x	-3	-2	-1	0	1	2	3
y	7	2		-2	-1	2	

- b Draw the graph of $y = x^2 - 2$. [2]

- c Find the values of x where the graph crosses the x -axis. [2]

- 2 a Draw the graph of $y = x^2 - 3x - 5$ for values of x from -2 to 5 . [5]

- b Use your graph to find the solutions of the equation $x^2 - 3x - 5 = 0$. [2]

6 Measures

Compound measures

Here is an exam question ...

Vivek cycled at 12 miles per hour for 12 minutes.
How far did he go? [2]

... and its solution

$$12 \text{ minutes} = \frac{12}{60} = 0.2 \text{ hours}$$

$$\begin{aligned} \text{Distance} &= \text{Speed} \times \text{Time} \\ &= 12 \times 0.2 \\ &= 2.4 \text{ miles} \end{aligned}$$

Now try these exam questions

- 1 A rectangle has dimensions 354 cm by 64 cm.
 - a Work out the area
 - i in cm^2 . [1]
 - ii in m^2 . [1]
 - b The dimensions were measured to the nearest centimetre. Write down the bounds between which the dimensions must lie. [3]
- 2 A block of wood is a cuboid measuring 6.5 cm by 8.2 cm by 12.0 cm.
 - a Calculate the volume of the cuboid. [2]

The density of the wood is 1.5 g/cm^3 .

 - b Calculate the mass of the block. [2]

More exam practice

- 1 A bicycle wheel has diameter 62 cm. When Peter is cycling one day, the wheel turns 85 times in one minute.
 - a What distance has the wheel travelled in 1 minute?
 - b Calculate Peter's speed, in kilometres per hour. [5]
- 2 The population of Denmark is 5.45 million. The land area of Denmark is $42\,400 \text{ km}^2$. Calculate the population density of Denmark. Give your answer to a sensible degree of accuracy. [3]

Bounds of measurement

Here is an exam question ...

When a ball is thrown upwards, the maximum height, h , it reaches is given by $h = \frac{U^2}{2g}$.

It is given that $U = 4.2$ and $g = 9.8$, both correct to 2 s.f. Calculate the upper and lower bounds of h . [6]

... and its solution

To find the upper bound of h , use the upper bound of U and the lower bound of g .

To find the lower bound of h use the opposite.

$$\begin{aligned} \text{Upper } h &= \frac{4.25^2}{2 \times 9.75} \\ &= 0.926\,282 \\ &= 0.93 \text{ to 2 s.f.} \end{aligned}$$

$$\begin{aligned} \text{Lower } h &= \frac{4.15^2}{2 \times 9.85} \\ &= 0.874\,238\,5 \\ &= 0.87 \text{ to 2 s.f.} \end{aligned}$$

Now try this exam question

- 1 A formula used in science is $a = \frac{v - u}{t}$
 $u = 17.4$, $v = 30.3$ and $t = 2.6$, all measured correct to the nearest 0.1. Find the maximum possible value of a . [4]

More exam practice

- 1 The volume of a solid metal cylinder is 600 cm^3 . The density of the metal is 15 g/cm^3 . Calculate the mass of the metal cylinder in kilograms. [3]
- 2 The population of Kenya is 2.6×10^7 , correct to 2 significant figures. The area of Kenya is 5.8×10^5 square kilometres, correct to 2 significant figures. Calculate the lower and upper bounds for the population density of Kenya. [6]
- 3 In 1968, a firm produced 1.2 billion litres of fizzy drink (correct to 2 significant figures). The volume of a standard swimming pool is 390 m^3 (correct to 2 significant figures). What is the greatest number of these swimming pools that could possibly be filled with this amount of fizzy drink? [4]

7 Percentage and proportional change

Repeated percentage change

Here is an exam question ...

- a** In a sale, a coat is reduced by £5. The original cost was £40. What percentage reduction was this? [2]
- b** Sian invested £5500 in a fund. 4% was added to the amount invested at the end of each year. What was the total amount at the end of the 5 years? [2]

... and its solution

- a** Fraction reduction = $\frac{5}{40}$
= 0.125
Percentage reduction = 0.125×100
= 12.5%
- b** Total amount = $£5500 \times (1.04)^5$
= £6691.59 (to the nearest penny)

Now try this exam question

- 1** A house went up in value by 1% per month in 2007. At the beginning of the year it was valued at £185 000. What was its value six months later? Give the answer to the nearest pound. [2 + 1]

More exam practice

- 1** Teresa invested £5000 at 4% compound interest for five years. How much was the investment worth after five years? [3]
- 2** A computer cost £899. It decreased in value by 30% each year. What was its value after
- a** 1 year? [2]
- b** 5 years? [2]

Finding an amount before a percentage increase or decrease

Here is an exam question ...

- a** Jenny used to go to the village hairdresser where a trim cost £4. The hairdresser left so she went to a salon in town. A trim there cost £30. What percentage increase is this? [2]
- b** At the town salon the prices have gone up several times. The last increase was 6%. Jenny now pays £37.10. How much did she pay before the last increase? [2]

... and its solution

- a** Increase = £26. Original was £4.
Percentage increase = $\frac{26}{4} \times 100$
= 650%
- b** New = 106% of previous. (1.06)

Previous is wanted so divide by 1.06

$$\begin{aligned} \text{Previous} &= 37.10 \div 1.06 \\ &= £35 \end{aligned}$$

Now try this exam question

- 1** The value of a car falls by 20% each year. A car which is 3 years old is worth £10 240.
- a** What was it worth last year? [2]
- b** What was it worth when it was new? [2]

More exam practice

- 1** All the clothes in a sale were reduced by 15%. Mark bought a coat for £68 in the sale. How much did it cost before the sale? [3]
- 2** Pensions were increased by 4% last year.
- a** Before the increase Brian's annual pension was £18 900. What was it after the increase?
- b** Howard's pension is now £13 000 per year. What was it before the increase? [5]
- 3** At Joe's Cars, all cars were reduced by 10%.
- a** What is the new price of a car that was £7000 before the reduction?
- b** David paid £4527 for a car after the reduction. What was its price before the reduction? [5]

Increasing or decreasing by a fraction

Try these exam questions

- 1 Bus fares are going to increase by $\frac{1}{8}$. The fare into town was £1.52. Work out the new fare. [3]
- 2 Due to bad weather, the attendance at the local football match was down by $\frac{2}{5}$ on the previous week's attendance. Last week the attendance was 4510. Work out this week's attendance. [3]

Repeated fractional change

Try these exam questions

- 1 Phil earns £280 per week. His pay is set to increase by $\frac{1}{12}$ each year. How much will Phil earn per week after 5 years? [3]
- 2 The local Council built 1100 houses this year. The amount of new building is expected to fall by $\frac{1}{9}$ each year. How many houses will they be building in 3 years time? [3]

Finding an amount before an increase or decrease

Here is an exam question ...

The Midland Railway has a special offer on some fares. This is the advertisement:

Fares Reduced
 $\frac{1}{3}$ off normal fare



You pay only £75.

What is the cost of the normal fare? [3]

... and its solution

Fare is reduced by $\frac{1}{3}$.

$$\begin{aligned} \text{Normal fare} &= \frac{75}{1 - \frac{1}{3}} = \frac{75}{\frac{2}{3}} \\ &= 75 \times \frac{3}{2} = \text{£}112.50 \end{aligned}$$

Now try these exam questions

- 1 Train fares are increased by $\frac{1}{9}$. After the increase, the fare from Manchester to London was £115. What was the fare before the increase? [3]
- 2 All the prices in Helen's shop are reduced by $\frac{1}{5}$ in a sale. The sale price of a dress is £72. What was the price of the dress before the sale? [3]

8 Standard form and using a calculator

Standard form

Here is an exam question ...

- Write these numbers in standard form.
 - 41 000 000
 - 0.000 062 9 [2]
- Work these out. Give your answer in standard form.
 - $(3 \times 10^{-9}) \times (7 \times 10^{11})$
 - $\frac{6 \times 10^4}{2 \times 10^{-4}}$ [4]

... and its solution

- 4.1×10^7
 - 6.29×10^{-5}
- $21 \times 10^2 = 2.1 \times 10^3$
 - 3×10^8

$$7 \times 3 = 21; -9 + 11 = 2$$

$$\frac{6}{2} = 3; 4 - -4 = 8$$

Now try this exam question

- 1 a Work out $(3.0 \times 10^4) \times (6.0 \times 10^3)$, writing the answer in standard form. [2]
 - A terawatt is 10^{12} watts. A power station produces 1.2×10^8 watts. Write this in terawatts. [1]

More exam practice

- Write the following as whole numbers or fractions.
 - 4^0
 - $9^{\frac{1}{2}}$
 - Evaluate $(3 \times 10^{-2}) \times (8 \times 10^5)$ giving your answer in standard form. [4]
- Evaluate $\frac{2^8 \times 2}{2^{-2}}$ expressing your answer
 - in the form 2^n .
 - in standard form correct to 3 significant figures. [3]
- A radar transmitter sends out a beam of radio waves at a frequency of 24 thousand million pulses per second. Write this figure in standard form. [1]
- The orbit of Halley's Comet means that it passes the Earth every 76 years. At its furthest point it is 35 Astronomical Units from Earth. An Astronomical Unit is 1.496×10^{11} m. What is the maximum distance of Halley's Comet from Earth? Give your answer in standard form, in metres. [2]

The efficient use of a calculator

Here is an exam question ...

Work out the following. Give your answers to 2 d.p.

- $4^2 + 3^2 - 2 \times 4 \times 3 \times \cos 20^\circ$
- $2.7 \times 10^{-6} \times 8.2 \times 10^{-4}$
- $\sqrt{3 + 5 \cos 40^\circ}$ [4]

... and its solution

- 2.45
- 2.21×10^{-9}
- 2.61

Now try these exam questions

- Find
 - $(3.6 \times 10^{-10})^{\frac{1}{2}}$ [1]
 - $(3.6 \times 10^{-9})^{\frac{1}{2}}$ [1]
- Work out these.
 - $6.3 \times 10^9 + 5.8 \times 10^{10}$
 - $\frac{(9.52 \times 10^{14})^2}{8 \times 10^{-3}}$ [3]

More exam practice

- Work out these.
 - $7.2 \times 10^7 - 8.6 \times 10^5$
 - $7.2 \times 10^7 \div 8.6 \times 10^5$
 Give each answer to a sensible degree of accuracy. [4]
- Find x if $\cos x = \frac{6.2^2 + 8.5^2 - 10.7^2}{2 \times 6.2 \times 8.5}$. [2]
- Find y if $\sin y = \frac{7 \sin 17^\circ}{8}$. [2]
- Work out these.
 - $6(\cos 27^\circ + \sin 27^\circ)$
 - $3 - 4 \sin 28.7^\circ$ [2]
- Work out $64^{\frac{3}{4}}$. [2]

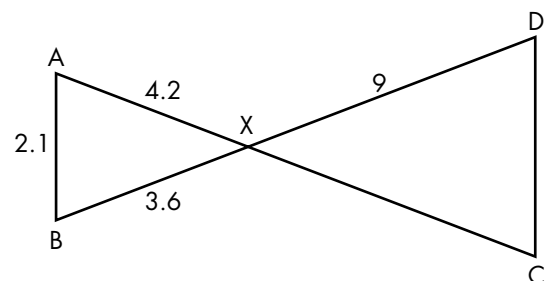
9 Similarity

Similar shapes

Here is an exam question ...

Triangles AXB and CXD are similar.

- Find the lengths of XC and DC. [7]
- Explain why AB is parallel to DC.



... and its solution

- $$\frac{XC}{4.2} = \frac{9}{3.6}$$

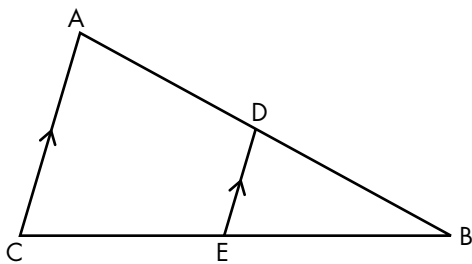
$$XC = \frac{9 \times 4.2}{3.6} = 10.5 \text{ cm}$$

$$\frac{DC}{2.1} = \frac{9}{3.6}$$

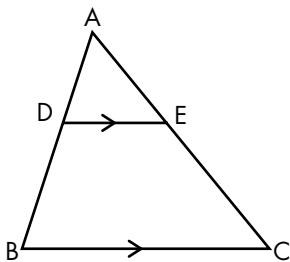
$$DC = \frac{9 \times 2.1}{3.6} = 5.25 \text{ cm}$$
- Angle BAX = angle DCX (corresponding angles in similar triangles). But these are alternate angles for AB and DC, making them parallel.

More exam practice

- 1 a Explain why triangle ABC is similar to triangle DBE.



- b Given that $DE = 1$ cm, $BE = 2$ cm and $EC = 4$ cm. Find the length of AC. [6]
- 2 a In triangle ABC, DE is parallel to BC. Prove that triangles ADE and ABC are similar.



- b D divides AB in the ratio 1:3. DE is 3 cm. Calculate the length of BC. [6]

The area and volume of similar shapes

Try this exam question

- 1 Three similar suitcases each have dimensions which are 25% greater than those of the next smaller suitcase.
- a The middle suitcase is 40 cm tall. How tall is the smallest suitcase? [3]
- b The middle suitcase has a volume of $32\,000\text{ cm}^3$. Calculate the volume of the largest suitcase. [3]

More exam practice

- 1 On holiday in France, we had two similar jugs on the table. One held 1 litre of water. The other held 50 cl of wine. The larger jug was 24 cm high. Calculate the height of the smaller jug. [3]
- 2 In Xian, China, you can buy solid scale models of the famous Terracotta Warriors. A model 16 cm high weighs 270 grams and has an armour plate of area 9 cm^2 .
- a Calculate the area of the armour plate area on a similar model of height 24 cm.
- b Calculate the mass of the same 24 cm model. [5]

10 Factorising

Here is an exam question ...

- a Factorise completely $6p^2 - 8p$. [2]
- b Factorise $x^2 + 3x - 4$. [2]
- c Factorise $2x^2 - 18$. [2]

... and its solution

- a $2p(3p - 4)$
- b $(x + 4)(x - 1)$
- c $2(x - 3)(x + 3)$

Now try these exam questions

- 1 Factorise completely $12p^2q - 15pq^2$. [2]
- 2 Factorise completely $6xy^2 - 12x^2y$. [2]
- 3 Factorise completely $6x^2 + 3x$. [2]
- 4 Factorise completely $ab^2 - 3a^2b$. [2]

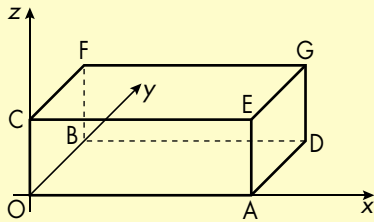
More exam practice

- 1 a Factorise completely $5x^2 - 20$. [2]
- b Factorise $x^2 - 9x + 8$. [2]
- 2 Simplify this expression as far as possible.
- $$\frac{x^2 + 3x}{x^2 + x - 6}$$
- [3]
- 3 Simplify $\frac{(x - 3)^2}{x^2 - 9}$. [3]

11 Three-dimensional geometry

Try this exam question

- 1 The diagram shows a cuboid.



A is the point (6, 0, 0).

D is the point (6, 4, 0).

C is the point (0, 0, 2).

- a Write down the coordinates of the following points.

i B [1]

ii F [1]

iii G [1]

iv the midpoint of AD [1]

- b Calculate the length of the diagonal OG. [3]

- c Calculate the angle that OG makes with the base of the cuboid OADB. [3]

Other types of proportion

Here is an exam question ...

From a point h metres above sea level the distance, d kilometres, to the horizon is given by $d \propto \sqrt{h}$.

When $h = 100$ m, $d = 35$ km. Find d when $h = 25$ m. [3]

... and its solution

$$d \propto \sqrt{h} \text{ or } d = k\sqrt{h}$$

$$35 = k\sqrt{100}$$

$$k = 3.5$$

$$\text{So } d = 3.5 \times \sqrt{25}$$

$$= 17.5 \text{ km}$$

Now try these exam questions

- 1 A bullet fired from a gun is slowed down by air resistance. The resistance is proportional to the square of the speed. If the resistance is 100 N when the speed is 300 m/s, find:

a the resistance when the speed is 600 m/s. [2]

b the speed if the resistance is 200 N. Give your answer to 3 s.f. [3]

- 2 When x takes a certain value, the value of y is 10. If this value of x is multiplied by 4, work out the value of y in each of the following cases.

a y is proportional to x [1]

b y is proportional to x^2 [1]

c y is inversely proportional to x [1]

- 3 y is inversely proportional to x^2 and $y = 9$ when $x = 2$.

a Find the equation connecting y and x . [3]

b Use the equation to find the values of x when $y = 1$. [2]

12 Proportion and variation

Direct proportion

Try these exam questions

- 1 On the label of a bottle of blackcurrant cordial it says: 'Mix 1 part blackcurrant with 3 parts water'. How much drink can be made from a 1.5 litre bottle of blackcurrant? [2]

- 2 A 5 litre tin of paint covers 60 m^2 . The amount of paint needed is directly proportional to the area to be covered. How much paint is needed to cover 15 m^2 ? [2]

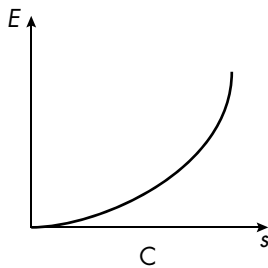
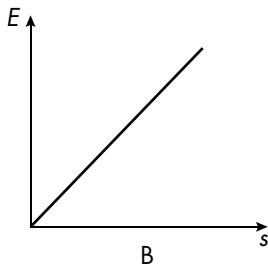
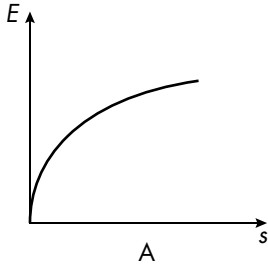
More exam practice

- 1 The variable y is directly proportional to x^2 . Given that $y = 75$ when $x = 5$, find the value of y when $x = 10$. [4]

- 2 The number of coins, N , that can be made from a given volume of metal is given by $N \propto \frac{1}{d^2}$ where d cm is the diameter. Given that 8000 coins with a diameter of 2 cm can be made from the volume of metal, how many coins with a diameter of 4 cm can be made from the same volume. [3]

- 3 The distance travelled by a car after the brakes are applied is proportional to the square of the initial speed. If it takes 12.5 m to stop when travelling at 50 km/h, how far will a car travel if its initial speed is 120 km/h? [3]

- 4 The energy, E joules, of a moving object is proportional to the square of the speed, s metres per second. When the object is moving at 8 metres per second it has 320 joules of energy.
- What is the speed of the object when it has 720 joules of energy? [2]
 - Which of these graphs, A, B, or C, represents the relationship between the energy, E , and the speed, s , of a moving object? [1]



- 5 The time of swing, T seconds, of a pendulum is proportional to the square root of the length, L centimetres, of the pendulum. A pendulum of length 64 cm has a time of swing of 1.6 seconds. Find the formula for T in terms of L . [3]
- 6 P is inversely proportional to the square root of Q . If $P = 12$ when $Q = 49$, find an expression for P in terms of Q . [3]

13 Graphs 2

Solving simultaneous equations graphically

Try this exam question

- 1 Solve these simultaneous equations graphically.

$$y = 3 - x$$

$$y = x^2 - 2$$

[4]

Using graphs to solve quadratic equations

Try this exam question

- a Draw the graph of $y = 2x^2 - 3x - 1$ [3]

- b Use your graph to solve the equations

i $2x^2 - 3x - 1 = 0$

ii $2x^2 - 3x = 2$

[2]

- c By drawing another graph on the grid, solve the equation $2x^2 - 2x - 3 = 0$ [3]

Drawing and recognising other curves

Here is an exam question ...

- a Complete the table below for $y = x^3 - 2x^2 + 1$.

x	-1	-0.5	0
y		0.375	

[1]

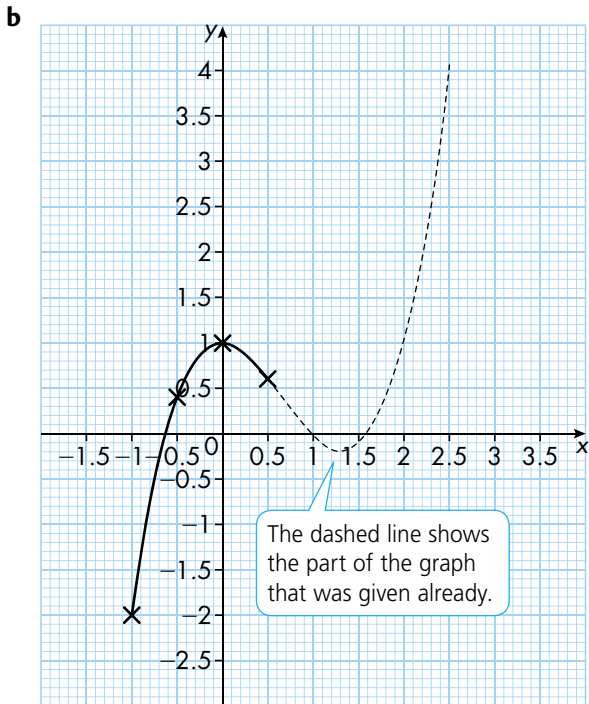
- b Part of the graph is drawn on the grid. Add the three points from the table and complete the curve. [2]

- c Use the graph to solve the equation $x^3 - 2x^2 + 1 = 0$. [2]

... and its solution

a

x	-1	-0.5	0
y	-2	0.375	1



c $x = -0.6, x = 1$ or $x = 1.6$. The solution is where the graph crosses the line $y = 0$.

Now try these exam questions

- 1 a Draw the graph of $y = \frac{2}{x}$ for values of x from -5 to 5. [3]
 b Use your graph to solve the equation $\frac{2}{x} = 0.8$. [1]
- 2 a Complete the table of values and draw the graph of $y = x^3 - 4x - 1$ for values of x from -3 to 3. [1]

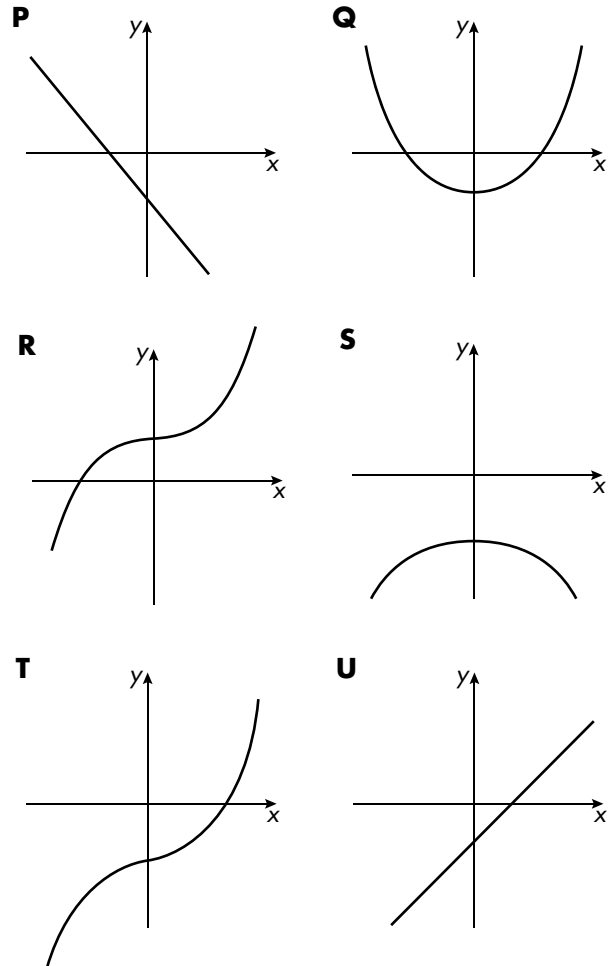
x	-3	-2	-1	0	1	2	3
y	-16		2	-1	-4	-1	14

- b Use your graph to solve the equation $x^3 - 4x - 1 = 0$. [3]
 c By drawing a suitable straight line on your graph, solve the equation $x^3 - 6x - 3 = 0$. [4]

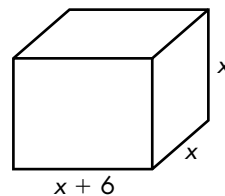
More exam practice

- 1 The diagram shows the graphs P, Q, R, S, T and U. State which of these graphs could correspond to each of the following equations.

- a $y = x^3 - 1$ [1]
 b $y = x^2 - 1$ [1]
 c $y = x - 1$ [1]



- 2 The volume, $V \text{ cm}^3$, of this cuboid is given by $V = x^3 + 6x^2$.



- a Copy and complete the table to draw the graph $V = x^3 + 6x^2$ for values of x from 1 to 6. [5]

x	1	2	3	4	5	6
V						

- b Use your graph to find the dimensions of the cuboid if its volume is 200 cm^3 . [1]

- 3 Three people start a rumour. Every hour each person who has heard the rumour tells one more person.
- a Complete this table for the number of people who have heard the rumour. [2]

Time (t hours)	0	1	2	3	4
Number of people (n)	3				

- b Draw a graph of t (horizontal) against n (vertical). [3]
- c Write down an equation connecting n and t . [2]

14 Quadratic equations

Here is an exam question ...

- a Write $x^2 + 6x + 2$ in the form $(x + a)^2 + b$. [2]
- b Hence state the minimum value of y on the curve $y = x^2 + 6x + 2$. [2]
- c Solve the equation $x^2 + 6x + 2 = 0$. [3]

... and its solution

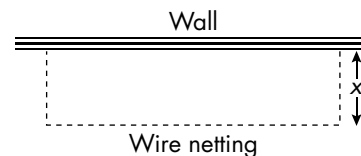
- a $x^2 + 6x + 2 = (x + 3)^2 - 9 + 2$
 $= (x + 3)^2 - 7$
- b The least value of $(x + 3)^2$ is 0, so the least value of y is -7 .
- c $x^2 + 6x + 2 = 0$
 $(x + 3)^2 - 7 = 0$
 $(x + 3)^2 = 7$
 $x + 3 = \pm\sqrt{7}$
 $x = -3 \pm\sqrt{7}$
 $x = -0.35$ or $x = -5.65$ to 2 d.p.

Now try these exam questions

- 1 a Factorise completely $5x^2 - 20$. [2]
- b i Factorise $x^2 - 9x + 8$. [2]
- ii Hence solve $x^2 - 9x + 8 = 0$. [1]
- 2 Solve the equation $2x^2 - 38x + 45 = 0$. Give your answers to 2 decimal places. [3]
- 3 a Write $x^2 - 12x + 2$ in the form $(x - a)^2 - b$. [2]
- b Hence find the minimum value of $x^2 - 12x + 2$. [1]
- c Solve the equation $x^2 - 12x + 2 = 0$. [3]

More exam practice

- 1 Solve these equations by factorising.
- a $x^2 - 6x + 8 = 0$ [3]
- b $2x^2 + 3x - 9 = 0$ [3]
- 2 a Multiply out and simplify the expression $(2x + 7)(3x - 6)$. [3]
- b i Factorise $x^2 + 6x$. [1]
- ii Solve the equation $x^2 + 6x = 0$. [1]
- 3 The length of a rectangle is y cm, the perimeter is 30 cm and the area is 55 cm^2 .
- a Form an equation in y and show that it can be simplified to $y^2 - 15y + 55 = 0$. [3]
- b Solve the equation $y^2 - 15y + 55 = 0$ to find the length and width of the rectangle. Give your answers correct to 2 d.p. Do not use a trial and improvement method. [4]
- 4 Bill is making a rectangular chicken pen against a wall. The other three sides will be made from wire netting. Here is the plan:



- The total length of wire netting is 22 m. The area inside the pen must be 60 m^2 .
- a Show that $x^2 - 11x + 30 = 0$. [3]
- b Solve the equation. [3]
- c Describe the size of the pen. [1]
- 5 Solve $x^2 - 4x - 6 = 0$. Give your answer in the form $a \pm \sqrt{b}$ where a and b are integers. [3]
- 6 a Solve $(x + 3)^2 = 49$. [3]
- b Hence solve the inequality $(x + 3)^2 < 49$. [1]

15 Simultaneous equations

Try this exam question

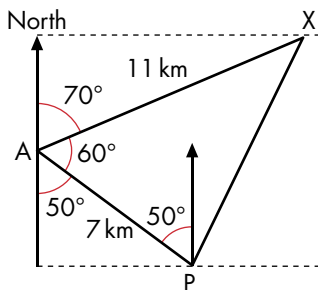
- 1 The curve $y = x^2 - 4x - 11$ meets the line $x + 2y = 8$. Show that $2x^2 - 7x - 30 = 0$ at the points of intersection. Hence find the coordinates of the points of intersection. [6]

16 Trigonometry

More exam practice

Here is an exam question ...

A ship sails from a port P a distance of 7 km on a bearing of 310° and then a further 11 km on a bearing of 070° to arrive at a point X where it anchors.



- Calculate the distance from P to X. [3]
- Calculate how far east of P the point X is. [2]

... and its solution

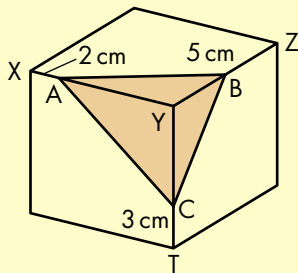
- $$PX^2 = 7^2 + 11^2 - 2 \times 7 \times 11 \cos 60^\circ$$

$$= 49 + 121 - 77$$

$$PX = 9.64 \text{ km}$$
- Distance east = $11 \sin 70^\circ - 7 \sin 50^\circ = 4.97 \text{ km}$

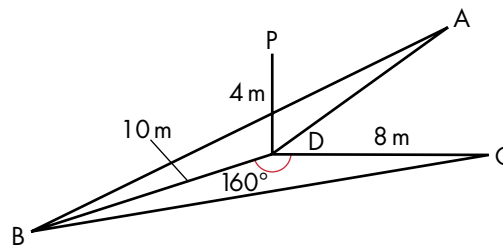
Now try this exam question

- X, Y, Z and T are four corners of a cube of side 10 cm. One corner is sliced off, as shown by the shaded portion, ABC. $XA = 2 \text{ cm}$, $BZ = 5 \text{ cm}$, $CT = 3 \text{ cm}$.



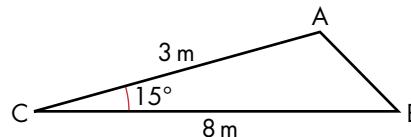
- Calculate the following lengths.
 - AB [2]
 - AC [2]
 - BC [2]
- Calculate the size of angle ACB. [4]

- To assist hang gliders, a large orange arrow is placed on the ground to show the wind direction.

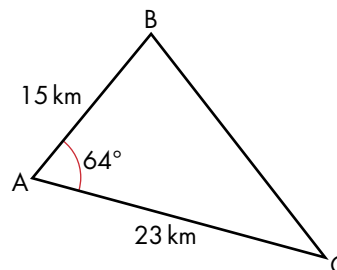


The triangles ABD and CBD are congruent. $BD = 10 \text{ m}$, $DC = 8 \text{ m}$, angle $BDC = 160^\circ$.

- Calculate the length of BC. [3]
- A vertical pole, DP, of height 4 m, with a flag at the top, is fixed at D and held by wires from A, B and C.
- Calculate the length of wire from A to P. [3]
- In triangle ACB, $AC = 3 \text{ m}$, $BC = 8 \text{ m}$ and angle $ACB = 15^\circ$.



- Calculate the length of AB. [2]
 - Calculate the area of triangle ABC. [3]
- The diagram shows the position of three ships A, B and C.

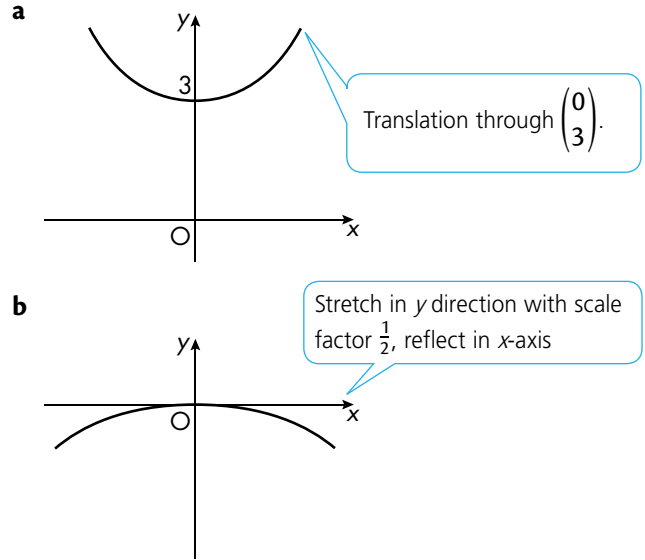


- Calculate the distance BC. [2]
 - An air-sea rescue search has to be made of the region inside triangle ABC. Calculate the area to be searched. [3]
- Sketch the graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$. [1]
 - Which other angle between 0° and 360° has the same sine as 30° ? [1]
 - Solve the equation $\sin x = -0.4$ for values of x between -360° and 360° . [2]

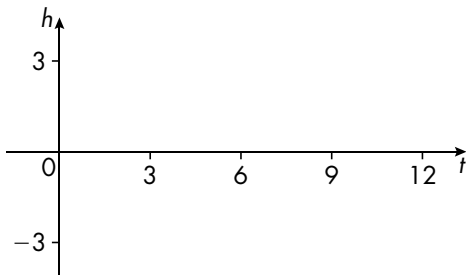
- 5 The diagram shows an indicator for the height of the water in a tidal river. At high tide the water is 3 m above zero and at low tide the water is 3 m below zero. The equation $h = 3 \sin(30t)$ can be used to find the height of the water at t hours after midnight.



... and its solution



- a Copy the axes and sketch a graph of $h = 3 \sin(30t)$ for $0 \leq t \leq 12$. [2]



- b On how many occasions in the 12 hours is the height of the water 2 m above zero? [1]

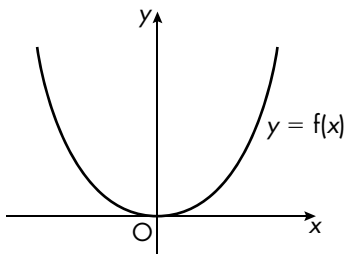
Now try these exam questions

- 1 a Sketch the graph of $y = x^2$. [1]
 b On the same diagram, sketch these graphs. In each case, describe the transformation.
 i $y = -x^2$ [1]
 ii $y = x^2 - 5$ [1]
 iii $y = \frac{1}{2}x^2$ [1]
 iv $y = (2x)^2$ [1]
- 2 On the same diagram, sketch the following graphs, labelling each graph clearly.
 a $y = \sin x$ [2]
 b $y = 3 \sin x$ [1]
 c $y = \sin(x + 90^\circ)$ [1]

17 Functions

Here is an exam question ...

The sketch shows the graph of $y = f(x)$.

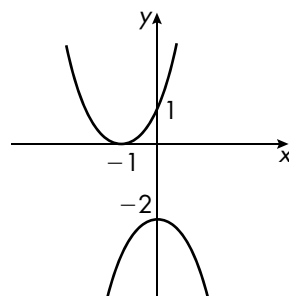


Sketch these graphs.

- a $y = 3 + f(x)$ [1]
 b $y = -\frac{1}{2}f(x)$ [1]

More exam practice

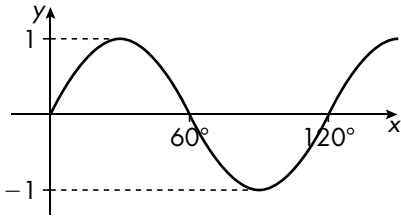
- 1 Describe the transformations that map $y = g(x)$ on to these functions.
 a $y = g(-x)$ [2]
 b $y = 4g(x)$ [2]
 c $y = g(\frac{1}{2}x)$ [2]
 d $y = 1 - g(x)$ [2]
- 2 The sketch graph show two functions transformed from $y = x^2$. Write down their equations. [3]



3 For each of these transformations of the graph of $y = f(x)$, write down its equation.

- a Reflection in the x -axis [1]
- b Stretch scale factor 3 parallel to the y -axis [1]
- c Translation $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ [1]
- d Translation $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ [1]

4 The sketch shows a function transformed from $y = \sin x$. Write down its equation. [2]

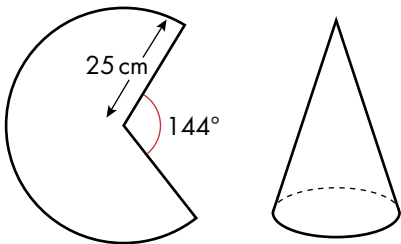


18 Length, area and volume

Arcs and sectors

Here is an exam question ...

A piece of card is cut from a circle of radius 25 cm as shown. The remaining card is folded so that the straight edges meet to make a hat with a circular base.



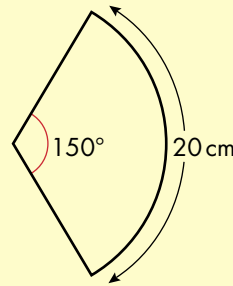
- a What is the surface area of the hat? [3]
- b What is the circumference of the base of the hat? [3]

... and its solution

- a Surface area = $\frac{216}{360} \times \pi \times 25^2$
= 1178.1 cm^2
- b Circumference = $\frac{216}{360} \times 2 \times \pi \times 25$
= 94.2 cm

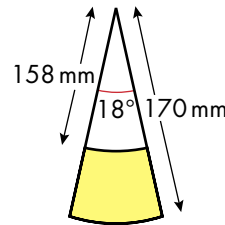
Now try this exam question

1 The diagram shows a sector of a circle. The angle is 150° and the arc length is 20 cm. Find the radius of the circle. [3]



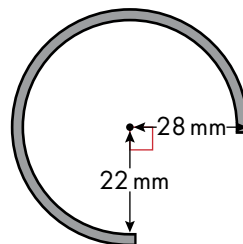
More exam practice

1 The diagram shows one sector of a dartboard. The yellow region is the 'double' zone.



Calculate the following.

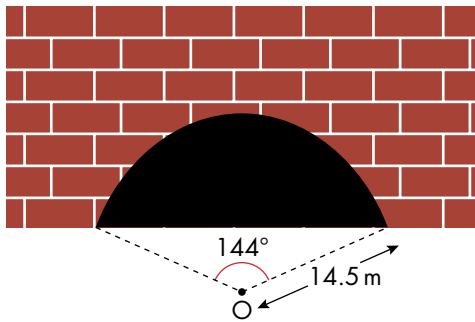
- a The perimeter of the yellow region. [5]
 - b The area of the yellow region [5]
- 2 A circlip is made by removing a piece of metal from a circular washer as shown in the diagram.



Calculate the following.

- a The area of the grey section [4]
- b The perimeter of the grey section [4]

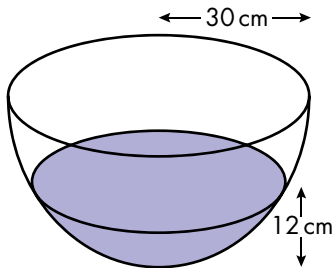
- 3 The diagram shows a tunnel entrance. The roof of the tunnel is part of a circle, centre O and radius 14.5 m. Calculate the area of cross-section of the tunnel. [8]



Pyramids, cones and spheres

Here is an exam question ...

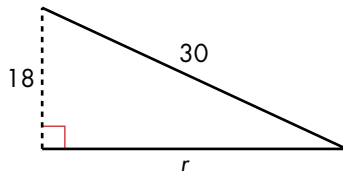
A hemispherical bowl has a radius of 30 cm.



- a Calculate the volume of the bowl. Leave your answer as a multiple of π . [2]
 b Water is poured into the bowl to a depth of 12 cm. Calculate the radius of the surface of the water. [4]

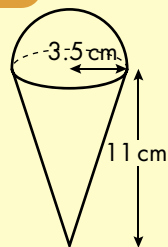
... and its solution

- a $V = \frac{1}{2} \times \frac{4}{3} \times \pi \times 30^3$
 $= \frac{1}{2} \times \frac{4}{3} \times \pi \times 27\,000$
 $= 2 \times \pi \times 9\,000$
 $= 18\,000\pi \text{ cm}^3$
 b $r^2 = 30^2 - 12^2$
 $= 576$
 $r = 24 \text{ cm}$



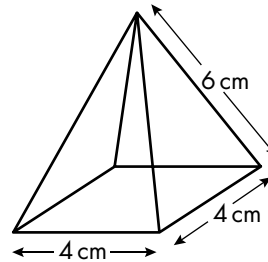
Now try this exam question

- 1 An ice-cream cone has height 11 cm and radius 3.5 cm. Ice-cream completely fills the cone and forms a hemisphere on the top of the cone. Neglecting the thickness of the cone, calculate the volume of ice-cream. [6]



More exam practice

- 1 The four slant edges of this square-based pyramid are 6 cm long. The four edges of the base are each 4 cm long. Calculate the volume of the pyramid. [7]



- 2 A cone has height 10 cm and base radius 4 cm. A smaller cone with height 5 cm is cut from the top of the cone. The part that is left is called a frustum.
 a What is the radius of the base of the smaller cone? [2]
 b Calculate the volume of the frustum. [4]

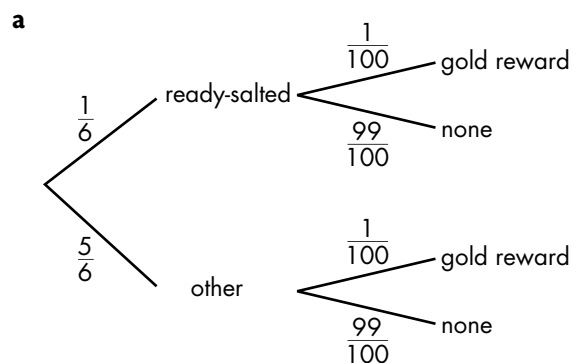
19 Probability 2

Here is an exam question ...

Pete likes crisps. Without looking, he picks a bag out of an assorted pack. There are 12 bags of crisps in the pack. Two of these bags are ready-salted. The manufacturers say that one bag in every 100 has a gold reward in it. This is independent of the flavour of the crisps.

- a Draw a tree diagram to show the probabilities. [3]
 b What is the probability that Pete picks a ready-salted bag with a gold reward in it? [2]
 c What is the probability that Pete picks either a ready-salted bag or one with a gold reward in it but not both of these? [3]

... and its solution

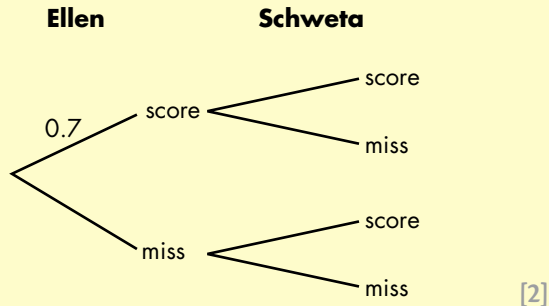


- b $P(\text{picks ready-salted with gold reward})$
 $= \frac{1}{6} \times \frac{1}{100} = \frac{1}{600}$
 c $P(\text{picks ready-salted or gold reward but not both})$
 $= \frac{1}{6} \times \frac{99}{100} + \frac{5}{6} \times \frac{1}{100} = \frac{104}{600} = \frac{13}{75}$

Now try this exam question

- 1 Ellen and Schweta are practising goal scoring in netball. When they each take one shot the probability that Ellen scores a goal is 0.7 and the probability that Schweta scores a goal is 0.8. These probabilities are independent.

a Copy and complete the tree diagram.



b Calculate the probability that, at their next attempt:

- i both Ellen and Schweta score a goal. [2]
 ii either Ellen or Schweta, but not both, scores a goal. [3]

- 2 Box A contains pens. 20% are red, 50% are blue and 30% are black. Box B contains pencils. 50% are red, 15% are blue and 35% are black. Jack takes a pen from Box A and a pencil from Box B at random.

- a Draw a tree diagram to show the possible outcomes. [3]
 b Work out the probability that he takes a red pen and a red pencil. [2]
 c Work out the probability that he takes a pen and pencil of the same colour. [3]

- 3 In this question express your answers as fractions in their lowest terms.
 There are 30 students in a class. The table gives information about boys and girls and whether they are left-handed or right-handed.

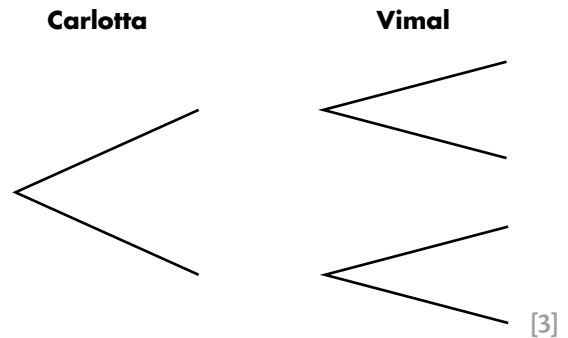
	Right-handed	Left-handed	Total
Boys	15	3	18
Girls	10	2	12
Total	25	5	30

- a A student is chosen at random. Given that the student is a girl, what is the probability that she is left-handed? [2]
 b Two students are chosen at random. What is the probability that they are both left-handed? [2]
 c Three of the girls are chosen at random. What is the probability that exactly two of them are right-handed? [3]

More exam practice

- 1 Carlotta and Vimal are both members of a walking club. Carlotta goes to 60% of the club's meetings. If Carlotta goes to a meeting, the probability that Vimal will go is $\frac{2}{3}$. If Carlotta does not go to a meeting, the probability that Vimal will go is $\frac{1}{4}$.

a Copy and complete this tree diagram using this information.



- b Using this tree diagram, find the probability that, for a randomly chosen meeting
 i both Carlotta and Vimal will go. [2]
 ii neither of them will go. [3]

- 2 James keeps his socks separately in a drawer. The drawer contains four red socks, five white socks, and six black socks. He dresses in the dark one morning and pulls out two socks without being able to see their colour. What is the probability that he takes out each of the following?

- a Two black socks [2]
 b Two socks of the same colour [3]
 c Two socks of different colours [2]

- 3 a Jake has a fair six-sided ordinary dice. He throws it twice. What is the probability that he throws a six both times? [2]
 b He throws the dice n times. Write down an expression in terms of n for each of the following probabilities.

- i He does not get a six on any throw [2]
 ii He throws at least one six [1]

- 4 Dee chooses three of these cards at random.

D E P E N D E N T

What is the probability that she chooses three cards that could be arranged to spell her name? [4]

20 Algebraic fractions

Here is an exam question ...

Michael drives 70 miles to work at an average speed of v miles per hour.

On the return journey he travels 5 miles per hour faster and takes $\frac{1}{4}$ hour less.

- a i** Write down expressions in v for the two journey times. [2]
ii Hence form an equation in v and show that it simplifies to $v^2 + 5v - 1400 = 0$. [3]
b Solve the equation to find v . [2]

... and its solution

- a i** $\frac{70}{v}$ and $\frac{70}{v+5}$
ii $\frac{70}{v} - \frac{70}{v+5} = \frac{1}{4}$
 $70(v+5) - 70v = \frac{1}{4}v(v+5)$
 $1400 = v^2 + 5v$
 $v^2 + 5v - 1400 = 0$
b $(v-35)(v+40) = 0$
 $v = 35$
 $(v = -40 \text{ is not possible.})$

Now try these exam questions

- 1** Use algebra to solve this equation. [7]
 $\frac{12}{3x+1} - \frac{5}{x+1} = 1$
- 2** Simplify these. [3]
a $\frac{2}{x-3} - \frac{1}{x}$
b $\frac{2}{x+1} + \frac{3}{x-2}$ [4]
- 3** Jane took part in a sponsored cycle ride. She cycled from her home town to Blackpool and back. The distance from her home town to Blackpool is 48 km. Jane cycled to Blackpool at an average speed of 12 km/h.
a Find the time she took to cycle to Blackpool. [2]
 Her average speed for the return journey was 8 km/h.
b Calculate her average speed for the whole journey. [4]

Sachin lives next door to Jane and took part in the same sponsored cycle ride. He cycled to Blackpool at an average speed of x km/h. He returned at an average speed of 5 km/h slower than on the outward journey. The total journey took him 8 hours.

- c i** Write down an equation in x and show that it simplifies to
 $\frac{6}{x} + \frac{6}{x-5} = 1$ [2]
ii Use algebra to find the solution to the equation in part **i**. [6]
iii Hence find Sachin's average speed for the journey from Blackpool to his home. [1]

More exam practice

- 1 a** Show that the equation
 $2x - 3 = \frac{3(x+1)}{x+4}$ can be rearranged to
 $2x^2 + 2x - 15 = 0$. [3]
b Solve the equation $2x^2 + 2x - 15 = 0$.
 Give your answer to 2 decimal places. [3]
- 2** Prove that $\frac{n+2}{n} - \frac{n}{n+2} = \frac{4(n+1)}{n(n+2)}$. [4]